O. TESTUDO

SPACE FOR ZENO*

Once upon a time, in the remote principality of Fallacia, there lived an ancient sophist who answered to the name of Zeno. In his youth, Professor Zeno had been something of an athlete; but as age took its toll, he found it progressively harder to reach the end of the race-track. Latterly, indeed, he had taken to competing against assorted reptiles: even so, his efforts had been marked by a humiliating absence of success. Outward frailty turned his thoughts inward, and he resolved henceforward to court immortality in the intellectual hippodrome.

For many years, Zeno had been depressed by the impoverished state of mathematics in Fallacia: the exponents of that divine art contented themselves with informal arguments and rough approximations, and in the name of rationality they ignored the real numbers. Even the geometers, Zeno noted with distaste, mocked at truth and based their computations on the false hypothesis that the value of π was 3.1416. 'How',

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he mused, 'might I inscribe my name in the Annals of Science? Let me amaze our insipid savants by excogitating the exact value of π. What discovery could be at once so noble and so beneficial? Tennis and golf will become rigorous skills; true-minted coins, when tossed, will slavishly obey the Law of Averages; and our philosophers will at last be able to reason in perfect circles.'

Ambition inspired action. Zeno took a large scroll of paper and began to write. He made a handsome start: 3.14159. . . But after several months of fruitful toil, he grew weary: the computations were simple enough, but he could foresee no early end to them. Weariness turned momentarily to despair when his old friend, Dr. Euclid, dropped by and casually informed him that his project was quite literally endless: however many digits he had jotted down, he would always find still more to jot; 'The decimal expansion of π', asserted that geometrical wizard, 'is infinitely long; and your life is finite. Why not collaborate with me on a book of *Elements* instead?'

But Zeno had no taste for textbooks. Moreover, his keen mind soon hit upon an antidote to Euclid's gloomy prognosis. 'After all', he reflected, 'I need only speed up my work a trifle to get it all done. Let me start again: I shall allow myself a comfortable ten minutes to compute the first digit in the expansion of π, and a generous five minutes for the second; the third can easily be obtained in $2\frac{1}{2}$, minutes, the fourth in $1\frac{1}{4}$, and so on. Surely in that way I shall have the whole thing done after no more than twenty minutes.' Euclid gave his *imprimatur* to the reasoning; and, placing a stopwatch in front of him, Zeno set to work again. Alas, his fingers had gone arthritic with age, and after $19^3/_8$ minutes his right hand was struck by an acute cramp.

Fortunately, however, Fallacian technology was highly developed. Zeno suppressed his natural disdain for things mechanical and telephoned his engineering colleague, Professor Archytas. Archytas laid aside his blueprint for a child's rattle and readily agreed to join forces with Zeno. The two men devoted themselves to the task of designing and constructing a device that might complete the work which human flesh had found too taxing.

And a splendid machine was soon put together. In essence, this π-machine, as they called it, was simple enough. It consisted of two parts, a small computer and a teleprinter. The computer was to take over from Zeno's fuddled brain: its function was to calculate the successive digits of π at an ever increasing speed; and it was programmed to take one minute for the first computation, $\frac{1}{2}$ minute for the second, $\frac{1}{4}$ minute for the third, and so on. The teleprinter was to take over from Zeno's arthritic fingers: its function was to type out the calculated digits; and it was geared to print the first digit one minute after its computation, the second digit ½ minute later, the third after another ¼ minute, and so on.

The two inventors resolved upon a public exhibition. In the presence of the whole nobility of the country, and of a large paying audience, Zeno himself uncovered the remarkable machine and pressed the starting-button. The computer hummed, and, after exactly two minutes, the teleprinter clacked out the digit '3'. Half a minute later, a hushed audience saw the digit '1' appear on the print-out sheet. Soon the machinery was running too fast for its particular motions to be discerned, and the print-out paper piled high at Zeno's feet. After two minutes there was a loud click, and the computer stopped; after a further minute a second click announced the cessation of the teleprinter's operations. Triumphantly, Zeno picked up the end of the print-out sheet and presented it, with a low bow, to the philosopher-prince of Fallacia, Gilbert the Metaphysical.

Gilbert sat back for a moment and tamped down his pipe. After a brief pause for reflexion, he passed judgement: 'Zeno, you are a fraud. This piece of paper you have given me is finite in length' — he held up the two ends to prove his point; 'and you can no more cr am infinitely many numerals on to a finite length of paper than hit a six at poker or swallow your *Pride and Prejudice*. The thing is impossible: this machine hasn't produced the goods.'

Gilbert's brisk argument convinced everyone present — and enraged the paying part of the audience. Zeno refunded the entrance money and sat in silent despondency. 'Perhaps', ventured Archytas, 'we have got not a ghost but a gremlin in our machine: shall we try again?' Without much hope, the two men returned to their laboratory and applied themselves to the invention of a second π-machine. Now Archytas had been reading Aristotle's *Physics* in bed; and he was suddenly struck by the pertinence of Aristotle's observation that the circle is in a sense infinite, for it has neither beginning nor end. There, he thought, lay the solution to his and Zeno's problem: substitute a continuous loop of paper for the long print-out sheet of the first π-machine; the loop, being infinite, would easily accommodate the infinite digits of π; and Gilbert would not be able to find two ends — or even one end — of the output paper in order to cast doubt on the length of the computation.

Π2, the new machine, was quickly constructed; but Archytas persuaded Zeno to try it out in private. The machinery was duly started; it hummed impressively; it clacked and whirred; and the print-out loop ran fast under the teleprinter keys. After three minutes, silence. Zeno removed the loop of paper. To his intense chagrin, he could make out no numerals on its surface: down the centre of the paper ran a broad band of black ink, where the machine had printed and overprinted — time and again — the successive digits of π. No doubt that black band contained all the digits of π, in their correct order; but who would believe it?

Zeno feared that they would again be accused of charlatanry if they exhibited Π2 to the general public. Archytas agreed, and suggested that they should add an erasing device to the teleprinter: each numeral would be rubbed out as soon as it had been printed, so that the loop would not be marked only with an illegible band of overprinting. But Zeno convincingly argued that the completely blank loop of paper which Archytas' device would yield was hardly likely to convince a sceptical audience that Π2 had really printed out every digit in π. Archytas remembered the reception given to his totally silent LP of the Music of the Spheres, and felt obliged to concur.

Nothing daunted, the two inventors set to work for a third time. The third, and final, π-machine involved many months of arduous research. Eventually, Archytas again hit upon the crucial idea: he thought up the notion of having a teleprinter with keys of ever diminishing size. To avoid the need for an infinitely large set of keys, he constructed a mechanical key-contractor, by means of which the keys on his teleprinter might be successively compressed *ad lib.* Π3 then consisted of a computer and teleprinter, together with a contractor.

The contraption was assembled, and Archytas carefully adjusted the dial of the contractor: immediately after the printing of the first digit, the contractor was to reduce the size of each key on the teleprinter by a half; and after each successive printing, each key was again to be reduced to half its size. In that way, Archytas reasoned, the teleprinter would require only a finite sheet of paper; for it would use less and less space for each of its successive operations. The whole decimal expansion of π could be printed on a sheet of foolscap.

This time, Zeno and Archytas were confident enough to call another public meeting — and to invite Prince Gilbert himself to start the machinery. A large and incredulous audience soon collected, and Gilbert pressed the button with a sceptical snort. The machinery rattled away; the paper moved slowly along; the keys of the teleprinter chattered at an ever higher pitch. After three minutes, again, silence. Zeno gently withdrew the paper and exhibited it to Gilbert.

The paper was printed with a wedge of type. To the left-hand edge of the wedge, the digits '3', '1', and '4' were clearly visible, the '1' half the size of the '3', the '4' half the size of the '1'. To the right of the '4' nothing was distinct to the naked eye; but Gilbert took the magnifying glass which Zeno held out to him, and grudgingly allowed that he could now discern more digits, a '1', a '5' and a '9', each proportionately smaller than its predecessor. Successively more powerful magnifiers revealed successively smaller numerals: to the right of every digit there appeared, on closer inspection, a further digit; no last digit could be observed, and even when the microscopical resources of Fallacia were stretched to their limit, the most powerful magnifier clearly showed that there were yet more numerals printed on the foolscap but too minute to be distinctly made out.

The inference was obvious: logic proved, and empirical testing verified, that Π3 had indeed computed and printed out the complete — and endless — decimal expansion of π. The audience applauded; Gilbert grunted his approval; Archytas returned to his music; Zeno published a short note in *Mind:* surely the Swedes would grant him a Nobel prize, and even the English would be persuaded to abandon their customary zenophobia.

Zeno was disappointed: he received some notoriety, but little adulation; indeed, foreigners were apt to dismiss his invention as a silly sophistry. A few solemnly linguistic philosophers took Zeno more seriously, and for a time the pages of *Analysis* were filled with little essays by aspirants for Oxford posts: Zeno's π-machines, they pointed out, were supposed to perform an infinity of tasks in a finite time; but a close attention to language would suffice to prove that supposition false.

Thus some cunningly urged that the word 'task' could only be applied to a performance lasting for some discernible span of time; that no finite stretch of time contained infinitely many discernible spans as distinct parts of itself; and that the π-machines, therefore, could not perform infinitely many *tasks* in a finite time. Others noted that 'infinite' means 'endless'; and they inferred that the π-machine could not complete, or come to the end of, its infinite tasks, even if it had the whole of eternity to run for.

Analytical philosophy had some vogue in Fallacia; and Zeno's colleagues feared that such analytical attacks might prove fatal to him. But Zeno himself remained serenely unmoved by the nice observations of the linguistical thinkers: he held first that their remarks were one and all false; and secondly that they were in any case impertinent to the claims of his machine. And sober judges sided with Zeno, suggesting that analytical philosophers of that vein should be banished to Grub Street or to the cellars of the Oxford English Dictionary.

Some years after the invention of Π3, a vast Congress of Philosophy was planned by the Fallacian Academy: eminent thinkers from every age and land were invited to attend. The official papers were received with the polite boredom customary at such gatherings; but in the taverns, between and after the official sessions, a lively set of altercations took place; and their chief subject was Zeno's marvellous machinery.

One of the discussions was opened by Socrates:

'My dear Zeno', he began. 'I have observed your machine and much admired its ingenious construction. Truly you are a man of Daedalian art; for your handiwork performs many and wonderful things.'

'Which things do you mean, O Socrates?'

'I mean, my friend, that the keys of your machine make ever shorter journeys over ever shorter spaces in ever shorter times — is that not so?'

'Yes indeed, for each "journey", as you call it, is precisely half the length of its predecessor, taking precisely half the time and covering precisely half the space.'

'And there are infinitely many journeys, but each journey is of finite size?'

'Surely.'

'Then there can be no smallest journey?'

'Evidently not; for each journey is followed by another of half its length.'

'And equally, there can be no smallest space, and no smallest time?' Zeno agreed.

'But tell me, my friend,' said Socrates, smiling quietly, 'are you not familiar with the *Little World Order* of our learned compatriot, Mr. Democritus? For in that work — which I hastily perused before my pupil Plato cast it into the furnace — Democritus says that matter is not infinitely divisible, but that there are minimal units of stuff, or atoms as he calls them.'

'I have indeed read the treatise, Socrates, and I admire its precocity; but what have atoms to do with the case?"

'Well', said Socrates, 'consider it this way: if there are minimal units of stuff, or atoms, may there not likewise be minimal units of space, or topons as I may call them?'

'Perhaps so.'

'And minimal units of time, or chronons?'

'No doubt.'

'And therefore also minimal journeys, or hodons?'

'Your analogies, as always, my friend, are clever; but where do they lead?'

'Well, Zeno, if there are indeed in nature topons and chronons and hodons, your ingenious machine will not, I fear, after all be capable of performing its tasks. For every journey its keys make over the paper will be an integral number of topons in length, and after a finite number of topons it will perform a journey, let us suppose, of one topon in length; and after that journey it must stop — for it can make no shorter journey.'

'I do not yet follow your reasoning', said Zeno.

'Consider it thus: suppose that the first journey your teleprinter takes along the paper is exactly 128 topons in length. Then the second journey will be of 64 topons; and the third of 32 topons; and so on. And the eighth journey, I think, will be just one topon in length. So that there can be no ninth journey at all; for the ninth journey would have to be half a topon in length, and that is impossible. Now the same argument goes for chronons, too; and it evidently makes no difference how long the first journey is supposed to be. Thus if your π-machine is to work, there can be no topons and no chronons, but time and space must be dense or infinitely divisible.'

Zeno sat silent, with a puzzled frown on his face; but before long his old friend, Euclid, came to the rescue:

'My dear Socrates', said Euclid, 'your talk of topons and chronons and the like only displays your lamentable ignorance of geometry — I am afraid that you will be refused admission to your pupil's Academy. For, as I prove in the tenth book of my *Elements*, there can, geometrically speaking, be no smallest magnitude or minimal length; and hence there can be no topons. For every magnitude of whatever size is divisible — indeed, every magnitude is infinitely divisible. If you deny that, you unseat the whole of mathematics: my *Elements* protects Zeno's π-machine from your amateur objections.'

Euclid's magisterial pronouncement was received with almost universal applause; and the dissident murmurings of a few stray Epicureans were drowned by Aristotle's loud assent. But adversity only stimulated Socrates to greater endeavours:

'You speak, O Euclid, of your mathematics; and surely your *Elements* will be reckoned by posterity as one of the glories of Greece. But think, is yours the only mathematics? is Euclidean geometry the only geometry?'

'At all events', said Euclid, 'it is the best geometry.'

'A young acquaintance of mine called Xenocrates', replied Socrates, 'who is a mathematician and a philosopher of no little ingenuity, ventures to take issue with you. He believes that there are indivisible lines, and he has constructed a geometrical system which, unlike your own, allows that there are indeed minimal lengths. Now perhaps he is right, and you are wrong. Or rather, consider it this way: you say, I think, that parallel lines, however far they may be produced, will never intersect?'

'I do.'

'Yet you are aware, my dear, that among the barbarians there are geometers of high repute who hypothesize that parallel lines may intersect at a point, and who raise new geometries on that hypothesis?'

'Barbarous geometries', growled Euclid, 'and suitable only for barbarous souls.'

'Let us grant as much', said Socrates; 'for I do not wish to defend the barbarian hypothesis. Rather, my argument is this: to ask whether the barbarian or the Euclidean geometry is correct is, I suppose, to ask whether this circumambient space in which we and our world are placed fits the barbarian or the Euclidean postulates better; it is to ask which geometry applies better to the physical world — if indeed either applies at all. Now *that* question is surely one which the science of geometry cannot herself decide. Neither your axioms, nor those of the barbarians, can themselves tell us whether or not they describe the topology of space; for that is not a geometrical question. And if our cobblers and shipwrights think that the space they work in is governed by the laws of Euclid's *Elements*, yet who can tell but that some clever philosopher of nature may not soon inform us that in reality space is curved, or some other such nonsense, and more fitted to a barbarian than to a Euclidean geometry? I do not say, my dear Euclid, that the barbarians are right and you are wrong. I only say that geometry, considered in and by herself, deals with abstract subjects of the mind's contemplation and not with the physical space that our bodies move about in. Space may indeed have your geometry; or it may have a barbarian geometry; you, as a true geometer, cannot say which geometry it actually has.'

'And what, pray', asked Euclid, 'has the barbarian hypothesis that parallel lines meet to do with the case of my friend Zeno?'

'Why this', said Socrates; 'just as geometry herself cannot decide the contest between Euclid and the barbarians, so geometry herself cannot adjudicate the dispute between Euclid and Xenocrates. You, my friend, have constructed an admirable geometry on the assumption that geometrical space is dense or continuous; the geometry of young Xenocrates admits indivisibles, and his geometrical space is neither continuous nor dense. Now neither you nor he can tell us, on the basis of your geometrical studies alone, whether physical space is continuous or rather — as I may put it — granular in structure. That is a problem for another discipline. If Zeno's machine works, then there are no topons, and physical space is dense, as is your geometrical space; but from the fact that your geometry assumes the denseness of space, you cannot, my good friend, infer that physical space is in actual fact dense. Your geometrical speculations, wonderful as they are, cannot provide any help for Zeno.'

At that point in the discussion, a large figure detached himself from the backgammon table where he had been busily forgetting the truths of his own philosophy. He delivered himself of an impatient admonition:

''Tis certain', he averred, 'that we have an idea of extension; for otherwise how should we talk and reason concerning it? 'Tis likewise certain that this idea, as conceiv'd by the imagination, tho' divisible into parts or inferior ideas, is not infinitely divisible, nor consists of an infinite number of parts: for that exceeds the comprehension of our limited capacities. Here then is an idea of extension, which consists of parts or inferior ideas, that are perfectly indivisible: consequently, this idea implies no contradiction: consequently, 'tis possible for extension really to exist conformable to it: consequently, the admirable contrivances of Mr. Zeno are mere chimaeras and phantasies of his brain, and can have no real existence.'

With that, Hume (for it was he) made to return to his gambling; but the curious company pressed about him, demanding some elaboration of his magisterial pronouncement. For, they urged, even if it is possible for space or extension to consist of indivisible parts or topons, it by no means follows that space does in reality consist of topons or that the celebrated π-machines are a sham. Hume's reply was peremptory.

'You allow', he said, 'that 'tis possible for space to exist conformable to our idea of it, and to consist of a finite number of indivisible parts; but if it be possible, 'tis certain that space does actually exist conformable to it; since its infinite divisibility is utterly impossible and contradictory.'

Hume would say no more; and for the most part his train of reasoning was received in mute astonishment. It took a Frenchman to reply to the Scot.

'*Le bon David*', he observed, 'informs us that his northern imagination cannot picture any pieces of space smaller than a given magnitude; and he would have us infer that no smaller magnitude can logically exist. But why should we attend to the deliverances of a fancy occluded by Scotch mist and dulled by Scotch whiskey? Has not M. des Cartes already proved to us that it involves a contradiction to say that there should be any atoms which are conceived as extended and also indivisible? For, as he says, we can certainly perceive it to be possible for an atom to be divided, since we suppose it to be extended. Now M. des Cartes' remarks on the nature of matter are not beyond dispute; but surely they are incontrovertible if we apply them to space: Mr. Hume's topons are, by definition, extended; but whatever is extended can be conceived to be divided into parts. And since the parts of a space are themselves spaces smaller than the parent space, there cannot conceivably be indivisible units of space or topons.'

There followed an argument on the limits of the human imagination: Hume had asserted that there was some magnitude than which no smaller could be imagined; his French opponent held that, on the contrary, he could imagine a magnitude smaller than any given magnitude, and that Hume's imagination had been stunted by his dour upbringing. The dispute rapidly became personal and unedifying; and none of the philosophers present seemed capable of solving it.

It so happened that the painter Apelles, who had been commissioned to take the official photographs of the congress, had overheard some part of the argument. He dared to hope that Art might triumph where Philosophy had failed; and he asked leave to address the company. The philosophers, always eager to mock an amateur, readily granted him audience; and he began thus:

'Gentlemen, let me humbly offer you the following observations. I hold it to be beyond dispute that whatever I can paint I can imagine — for how can I picture on canvas what I cannot picture in my mind? It follows, I think, that if there is no smallest paintable magnitude, there is no smallest imaginable magnitude. Consider, then, how I represent magnitude in my own work — suppose, if you will, that I must portray first a short man and then a tall man. In each case I shall produce an image or representation of a human form; and each image will, of course, have a certain size — for it will occupy a certain area of my canvas. But the size of the image of the short man does not represent his low stature, nor does the size of the image of the tall man represent his greater stature. For, evidently, the image of the short man (if he is a wealthy patron) may be twice the size of the image of the tall man.

'Why, then, is my first picture an image of a short man? How can a relatively large image represent a relatively small man? Plainly, the stature of the short man may be indicated by the comparative sizes of his image and the image of the tree against which he is nonchalantly leaning; or I may, if I please, casually sketch into the background of my picture some ruler or measuring device; or I may simply label my picture: *Portrait of a Short Man*. None of that signifies; for my point, gentlemen, is an exceedingly simple one: an image that is, say, two feet high may represent a man of six feet or of five feet or of four feet — or a Lilliputian of four inches. It may represent a man of whatsoever stature I please. Now since this figure' (here Apelles sketched a quick caricature of Hume) 'may represent a man of whatever size, I infer that there is no limit to the power of my pictorial art, and that there is no smallest paintable magnitude. It follows that there is no smallest imaginable magnitude — and I submit that your dispute about the imagination is solved.'

The company was much abashed by those sane and convincing remarks; and Socrates made a mental note to allow at least one artist into Callipolis. But it takes more than Art to silence philosophers: after a short interval the characteristically English voice of Professor Pigstream was heard.

'What Apelles says is all very true. But I don't think that it advances our discussion. The concept of the imagination, with which Mr. Hume and his friends are so concerned, is not, after all, of any particular relevance to logic. We wish to know whether or not space can be divided *ad infinitum*: Mr. Hume and the disciple of M. des Cartes have each told us something about their own imaginative powers; and Apelles has just sided with the French. But the imaginable and the possible are really perfectly distinct. Some of Apelles' own successors have managed to depict impossible objects, and any child who makes an arithmetical error in effect conjures up in his imagination something that cannot possibly be so. Conversely, from the fact that a thing is possible, we can hardly infer that it is imaginable; for surely there are many possibilities — indeed, many actual facts — whose intricate nature is too much for our human imaginations. (Think of the things theologians customarily say about God.) Now our present question concerns possibility — so the limits of the imagination are of no real account.

'In fact', Pigstream continued, 'we should be appealing not to the imagination but to logic; and I can easily offer you a simple little argument which will show the impossibility of topons and the necessity of continuously divisible space'. (At this point, Socrates was observed to smile gently to himself, and some of his neighbours anticipated another bout of dialectic.) 'My argument', said Pigstream, 'relies exclusively on the logical or conceptual properties of our notion of spatial magnitude. Let us hypothesize that there *are* topons, and reduce that hypothesis to absurdity. Let this be a topon' — he drew a short line with his finger through the spillage on the table in front of him — 'and let us designate its end-points A and B. We shall, I suppose, agree that A is a distinct point from B; for if it were not, our topon would have no magnitude at all, but be a mere point. But we shall also agree that there is a third point, C, *between* A and B; for if there were not, A and B would be contiguous, and no two points can be contiguous. Now consider the interval AC: plainly, that interval determines a distance (for C is not identical with A); equally plainly, the distance determined by AC is shorter than that determined by AB (for C is between A and B). Consequently, the distance determined by AB is not, after all, the *shortest* possible spatial distance. But, by hypothesis, it *is* the shortest possible distance. We have reached a contradiction, and we may properly conclude that our original hypothesis is false and that there are no topons.'

Socrates had been visibly chafing during that long monologue; but he restrained himself for several minutes after Pigstream had finished speaking before again entering the intellectual lists.

'O Stranger', he began, 'your argument is both subtle and ingenious; nor am I sure that I have understood all that it contains. But I do not wish to earn a reputation for cristical quibbling, and I shall not ask you to repeat what you have said or examine more closely the various steps which your reasoning has taken — although I fear that you may have committed the error which some of our sophisticated dialecticians call 'begging the question'. Instead, let me ask you something simpler. Or rather, let us consider this line, XY' — and he drew a line on the table somewhat longer than Pigstream's beery AB. — 'Anyone who thinks that there are topons in nature will hold, I suppose, that XY can be divided at certain points and nowhere else?'

'Indeed.'

'Then suppose that XY consists of precisely four indivisible topons. Now I, O stranger, might be tempted in my ignorance to believe that XY might actually be thus made up of four topons; but you, if I grasp your meaning, hold that that is not so, but that as a matter of logic the line XY is divisible at more than three points.'

'That is so, Socrates', said Pigstream.

'For, you say, within the segment XZ, which I might foolishly hold to mark off the first topon in XY, there must be an intermediate point, and that point will divide XZ into two distances each of which is *less* than XZ and therefore less than our putative topon?'

'Exactly.'

'But what do we mean when we assert that there is some given point on a given spatial line? — Look at it this way: If there are topons, then any two-dimensional worm that occupies the line XY must be an integral number of topons in length — 1 or 2 or 3 or 4 topons, and nothing else. And any worm that moves along XY must move an integral number of topons at a time; for it cannot come to a stop between X and Z — for there is nowhere between X and Z at which it could come to a stop.'

'That seems right, Socrates', agreed Pigstream. 'And I may add that when I say that there are *no* topons, I mean to deny, first, that all bodies must be an integral number of topons in length; and, secondly, that all movements must proceed for an integral number of topons. I deny that your worm must have just one of the four lengths you describe, and I deny that it must move in the jerky fashion your account implies. In short, I do not believe that all lengths, whether of bodies or of motions, are precise multiples of some given length.'

'And now, O stranger, let me confess my difficulty. Your logic, you say, is enough to assure you that there are no topons; for it can prove that there are indeed points of space on my line XY *between* X and Z. But what a strange and incredible potency your logic must possess. My friend Cebes here is, I suppose, a head shorter than I am; and your logic, I think, will claim to tell me that it is possible for there to be men intermediate in height between Cebes and myself.'

'Perhaps it will.'

'But tell me, by the dog, how that can be.'

'Is that not an easy request, Socrates? For surely there *is* an intermediate height? and surely your friend, or someone else, might attain that height?'

'Now that, my dear, is just what I deny. Or rather, I deny that your logic can show that there are such heights and such growths. For the heights which men may attain are, I suppose, determined by the science of physiology which young Aristotle is so busily pursuing; and for all you or I can tell, science may prove that no man can attain a height intermediate between Cebes and me — do you not agree?'

He agreed.

'And logic, I suppose, cannot dictate an answer to physiology?'

'Indeed it cannot', said Pigstream. 'Logic may assert that certain answers are possible and others impossible; it may assert the consistency or the inconsistency of certain physiological suppositions. But it cannot answer those questions itself, nor can it determine what heights a man may in actual fact attain.'

'Well then, my friend. . .'

'Well what, O Socrates? I do not see how your humble talk of human growth bears upon the theory of space we are discussing.'

'Well then, my friend,' resumed Socrates, unperturbed, 'do we not say that there is such a science as physics?'

'We do.'

'And may not physics one day confirm the hypothesis of dear Democritus, telling us that all physical bodies must be composed of certain minimal substances?'

'It may.'

'And that the energy they possess must come in certain determinate parcels?'

'Yes.'

'And that the distances they traverse must be composed of certain determinate stretches? and that the magnitudes to which they extend must be made up of certain determinate sizes?'

'I suppose so.'

'But in that case, my friend', said Socrates, 'will not the wonderful science of physics have shown that there are indeed topons in nature? Now your logic may tell us something about the consistency of the assertions made by physics (unless indeed some clever logician should think to alter your famous logic in the light of the discoveries of physics), but logic cannot possibly dictate to physics or tell us whether there are topons in space or not.'

'It seems so.'

With that, the philosophical affairs to the evening were terminated, and the participants retired each to his own pleasure. Professor Zeno remained alone, trying to compose his thoughts. 'It now appears', he muttered to himself, 'that my π-machines will only function if I set them to work in a space that is dense — a space that does not consist of indivisible spaticules or topons. Well then, is the space we Fallacians inhabit dense, or is it so to speak granular in structure? can it be divided *ad infinitum*, or will division eventually come up against indivisible *minima*? The royal science of geometry, it seems, is impotent to decide that question; for geometry, considered in itself, is an abstract and unrea artl which cannot inform its votaries about the properties of real physical space. The human imagination, on the other hand, tells us a great deal; but it speaks with a forked tongue — it tells Mr. Hume one thing and M. des Cartes another. And in any case, it is the voice of phantasy and can give us no reliable information about real possibility. Logic is more potent than geometry and less fickle than imagination; but while it will not misguide us, it will not guide us either. *A priori* thought may determine what structures space can consistently possess; but it cannot determine what structure our space does in fact possess. What then remains? Empirical science — physics. It is the physicists, those philosophers of the *a posteriori*, who must discover — by deduction from high theory or by lowly and microscopical examination — whether matter moves smoothly or by what they may call quantum jumps, and whether the space in which matter moves is dense or granular.'

Next morning the foreign philosophers called to bid a crapulous farewell to Zeno. They found him at work in a makeshift laboratory: he was, he told them, just perfecting a new mechanical device for investigating the topology of space. He showed them an expanding motor-car jack, fitted with a finely geared handle. The first complete turn of the handle would cause the jack to expand by six inches; the second complete turn would expand the jack by three inches; the third by 1 $\frac{1}{2}$; the fourth by $\frac{3}{4}$. . .

NOTE: A tattered MS of the piece above printed was discovered in a dusty corner of the Library of Oriel College, Oxford. Its origin is unknown; but the discussion it reports evidently influenced the work of Adolf Grünbaum (see his excellent book on *Modern Science and Zeno's Paradoxes*, London, 1968). Readers who wish to verify the historical accuracy of the Fallacian narrative could turn to vol. 1, chh. XI - XII of Jonathan Barnes' recent work on *The Presocratic Philosophers* (London, 1979).