# Instrumental genesis of students' comparison strategies in a digital environment of dynamic Geometry

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**ABSTRACT**

*This paper reports on how a different learning approach in the context of digital environment of dynamic Geometry contributes to the construction of a variety of comparison strategies by students, according to the theory of instrumental genesis. We explore how the tool is shaping students' actions and simultaneously how students shape the tool by attributing to it new uses. In our experiment participated 48 pupils (13-14 years old). From the qualitative analysis of the data comes that the students have dynamically exploited the tools of the educational software Cabri-Geometry II and approached the comparison of lengths in many different ways, most of them unusual for school practice.*

**KEYWORDS**

*Instrumental genesis, Cabri-Geometry II, comparison, secondary school*

**RÉSUMÉ**

*Cet article explique comment une approche différente dans le contexte des environnements numériques de la géométrie dynamique contribue à la construction de diverses stratégies de comparaison par les élèves, conformément à la théorie de la genèse instrumentale. Nous explorons la manière dont l'outil façonne les actions des étudiants et, simultanément, la manière dont les étudiants façonnent l'outil en lui attribuant de nouvelles utilisations. Dans notre étude ont participé 48 élèves (âgés de 13 à 14 ans). L'analyse qualitative des données montre que les élèves exploitent de manière dynamique les outils du logiciel pédagogique Cabri-Géomètre II et abordent la comparaison des longueurs de différentes façons, la plupart inhabituelles pour la pratique scolaire.*

**MOTS-CLÉS**

*Genèse instrumentale, Cabri-Geomètre II, comparaison, école secondaire*

## INTRODUCTION

### *The theoretical problem*

In the Greek curriculum of Junior High school (students of 12-15 years old) comparison is mainly approached numerically or algebraically. In our research, we have considered appropriate to design a task in order to permit a systematic study of students' performance in comparison situations in the environment of Cabri-Geometry II, so that students can create their own cognitive representations regarding comparison, before they deal with the mathematical formalism of the algebraic comparison rule.

In what follows, we report an experiment analyzing students' performance in a 'multiple solution task' (Balomenou & Kordaki, 2009), concerning comparison as a function of the mediating digital tool used to solve this task by the students. According to literature, tools that students have at their disposal shape their solution strategies to a given task (Balomenou, Komis & Zacharos, 2017; Rabardel, 1995; Trouche, 2004). The use of a tool is never neutral (Rabardel & Samurçay, 2001). On the contrary, it originates a re-organization and mobilization of students' cognitive structures concerning the notion under investigation.

## THE CONCEPTUAL FRAMEWORK

### *The theory of instrumental genesis*

The instrumental genesis is the process of an artifact becoming an instrument in the hands of a user through two simultaneous processes: *instrumentalization* and *instrumentation* (Trouche, 2004). It is the dialectic by which learner and artifact are mutually constituted in action (Trouche, 2004; Vérillon & Rabardel, 1995; Vérillon, 2000). The affordances and the constraints of the tool influence the student's problem-solving strategies (instrumentation), while the student's knowledge guides the way the tool is used and, in a sense, shapes the tool (instrumentalization). In short, the student's thinking is shaped by the artifact, but also shapes the artifact (Hoyles & Noss, 2003).

In our study, we used the theory of instrumental genesis (Rabardel, 1995) as a means to analyze how students learn while they interact with digital tools of multiple representations, by examining students' comparison strategies that they develop while interacting with the artifact Cabri Geometry II and the underlying concept of comparison, in the context of a specially formulated "multiple solution" student task.

### *The Dynamic Geometry Environment Cabri-Geometry II*

Cabri-Geometry II is a Dynamic Geometry Environment reflecting the dynamic, interactive character of the computer medium for the conceptualization of mathematical concepts and properties, by supporting simultaneously multiple interconnected dynamic representations of geometrical, graphical, numerical, algebraic and symbolic aspect (Laborde, 1990; Laborde & Capponi, 1994).

Therefore, we considered appropriate to exploit Cabri-Geometry II in order to approach the notion of comparison, in terms of visualization, exploration and dynamic handling of geometrical constructions. The abundance of Cabri-tools, in combination with the dynamic visual output provided, can act as intrinsic feedback in inspiring students in the creation of several comparison strategies, unusual in the school practice. In this way learners have the opportunity to personally control their learning visually and symbolically (Assude & Gelis, 2002; Assude, 2005; Healy & Hoyles, 2001).

### *The research question*

In this article, the didactical issue under consideration is the investigation of students' strategies regarding comparison of lengths: "*How are students' strategies regarding comparison of lengths influenced by the comparison tool they use and how is the artifact modified by the uses that the students ascribe to it?"*.

## METHODOLOGICAL FRAMEWORK

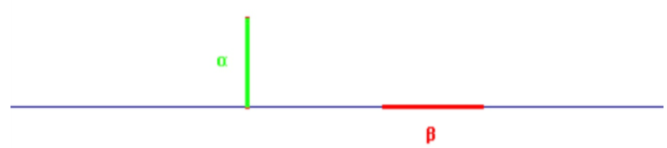
### *The overview of the study*

This qualitative research (Cohen, Manion & Morrison, 2007) reports a case study that took place in the computer laboratory of a Greek Junior High School. Forty-eight students of the 2nd grade of Junior High school (13-14 years old) participated in this learning experiment. These students worked in four groups of twelve students each, in the school computer laboratory. Students in each group worked individually in the school computer laboratory by exploiting Cabri-Geometry II and by taking notes in specially formulated worksheets simultaneously, in order to perform the given task, which was created by the researcher of the study and simultaneously teacher of the class. Each student participated for about one hour to complete the task.

### *The task*

Students were asked to exploit several Cabri-tools in order to compare the length of two given segments in the interface of Cabri (constructed by the researcher in such a way that they had a constant unchanged length; in this way it was possible to be moved through "copy & paste" or by using the "drag-mode" operation in order to change the orientation of the segments etc.) in a situation where optical perception did not suffice (Figure 1).

**FIGURE 1**




*Comparison of the 2 line segments - Snapshot from Cabri screen*

Students were also asked to simultaneously write down (on their worksheets) a short description of each of their solution strategies that they were about to implement on Cabri environment. Students were encouraged to create as many comparison strategies as possible. A familiarization phase using the functionalities and tools of Cabri took place before these students commenced the main study.

### *The overview of the analytic procedure*

The data sources of our intervention are the digital Cabri-files with students' actions, the video recordings of students' actions in the interface of Cabri, the students' worksheets and the fieldnotes of the researcher. The whole experiment has been recorded by using the software *Camtasia*. Forty-eight hours of students' actions have been recorded in total during the experiment.

An integrative qualitative analysis of students' digital and written work was carried out (Cohen et al., 2007). The qualitative data of our intervention were analyzed with respect of students' actions and instrumented techniques in the environment of Cabri. In order to answer our research question, we have defined as the unit for our analysis to be *the activity of each student throughout the process.* As variable of our analysis we have defined *the comparison of segments*. As values of the variable in our analysis we have defined *the comparison strategies developed by students.*

Each individual's solution strategies to the given task on the Cabri environment were identified and reported. These strategies were analyzed in terms of students' approaches regarding the notion of comparison through the construction of instrumented techniques regarding comparison. In the next stage, the focus was on the classification of students' strategies. The criterion for this classification has also been students' instrumented techniques. Finally, the role of the provided tools in the construction of these strategies has been studied according to the theory of instrumental genesis.

## RESULTS

The analysis of our data with regard to our *research question* about students' comparison strategies showed that all students of the research were actively involved in the task and created at least three solution comparison strategies each. The total number of the comparison strategies that they have been constructed by the 48 students of the study are 325 strategies. Students were encouraged by the researcher to create as many comparison strategies as possible by trying several tools. These strategies were identified and grouped into three categories (Table 1): dynamic geometrical approach (C1), numerical approach (C2) and mixed approach (C3). In the dynamic geometrical approach (C1) fall comparison strategies based on the exploitation of geometrical concepts and shapes without any kind of numerical measurements. In the numerical approach (C2) fall comparison strategies based on numerical measurements. In the mixed approach (C3) fall comparison strategies that combine geometrical concepts and shapes with numerical measurements.

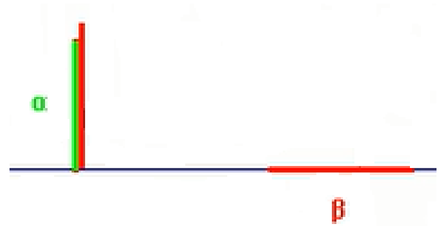
**TABLE 1**

*Categories of students' strategies regarding the comparison of lengths in the context of Cabri*

| **Categories of students' comparison strategies in Cabri ( 3 - 13 strategies per student)** | **Frequency (sample size: 48 students)** |
|---|---|
| C1: dynamic geometrical approach | 234 |
| C2: numerical approach | 55 |
| C3: mixed approach | 36 |
| **Total** | **325** |

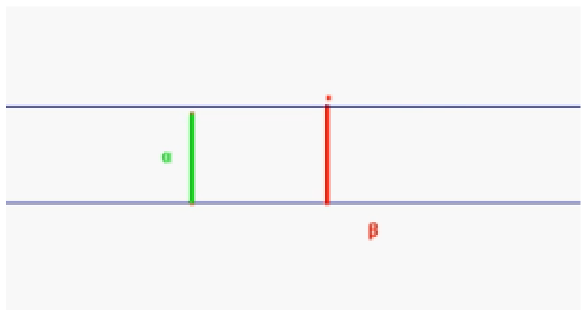
The most frequent comparison approach is the geometrical approach (category C1). Indeed, 234 out of 325 strategies are of geometrical nature. Two representative examples of the geometrical comparison approach are presented in the following figures (Figure 2 and Figure 3).

**FIGURE 2**




*Direct visual comparison through 'copy-paste'*

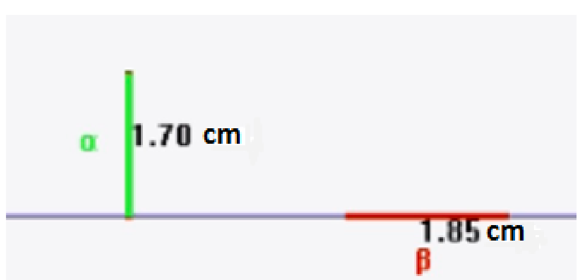
**FIGURE 3**




*Visual comparison by exploiting a parallel line from the upper edge of the red segment*

The second most frequent comparison approach was the numerical approach (category C2). In this category falls a significant smaller amount of comparison strategies (in comparison to the number of strategies that fall in the geometrical approach). Indeed, 55 out of 325 strategies are of numerical nature, despite the fact that comparison through numerical measurement is the usual school comparison approach. At this point, we should remark that the geometrical nature of Cabri seem to affect both students' action and thinking (instrumentation process -Trouche, 2004).
Two representative examples of the numerical comparison approach are presented in the following two figures (Figure 4 and Figure 5).

**FIGURE 4**




*Comparison of segments through measurement of their lengths*

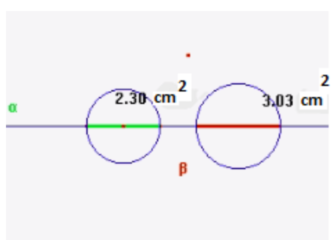
**FIGURE 5**

*Comparison of segments by using the grid dots as a unit of measurement*

Moving to the third comparison approach, we should notice that 22 students created 36 strategies that fall into the mixed approach (category C3), which combine geometrical comparison and comparison with measurements.

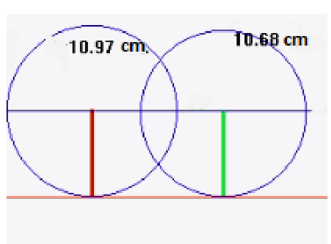
Two representative examples of the mixed comparison approach are presented in the following, in which the 'circle' tool seemed to be the most prevailing tool in students' constructions also in this category of strategies, enabling students to compare one-dimensional sizes in a particular way and unusual for school practice: transforming the comparing sizes into comparable geometric shapes where the visual comparison is obvious. As a matter of fact, this strategy has acted as a "zoom in" technique in order to make the comparison more visible. Students that created strategies in this category, have also utilized additional tools to measure circular disk area (Figure 6) or circle circumference (Figure 7) either to verify - confirm their conjecture of their visual comparison through this particular strategy for comparison through magnitude or in order to make more accurate estimations after using their visual perception.

**FIGURE 6**




*Comparison of lengths through the comparison of the area of circles having the segments as diameters*

**FIGURE 7**




*Comparison of lengths through the comparison of the circumference of the circles having the segments as radii*

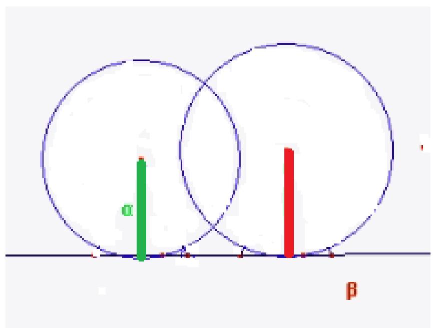
In what follows we focus on a very interesting and representative students' comparison approach that gives answer to our research question by revealing aspects of instrumental genesis while students compare the given in Cabri-Geometry II in the context of the given task.

***The tool for circles***

In our research 47 out of 48 students, almost all of them, have constructed 67 comparison strategies by using the 'circle' tool. This tool was utilized in two ways:

1) *As a magnification tool.* Students used the segments under comparison as radii or diameters of circles. Then, they visually compared the area of two circles and concluded that the biggest segment corresponds to the biggest circle (Figure 8).

**FIGURE 8**

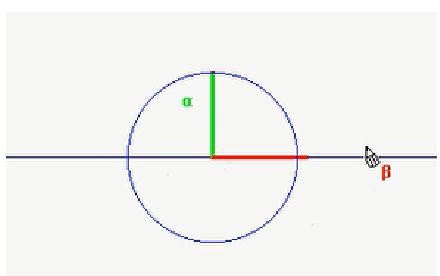



*Using the two line segments as radii of circles and visual comparison of the circle areas*

So, the *'circle'* tool was exploited as a magnification tool, in an attempt to make visual comparison obvious. This comparison technique identifies new use that students of the study have given to the 'circle' tool within this activity (instrumentalization process - Trouche, 2004).

2) *As a bounding tool.* Some students placed the segments vertical to each other or successively and then used one of them as radius and constructed a circle. Then they observed if the other segment protruded or not from the circle perimeter (Figure 9).

**FIGURE 9**




*Constructing a circle with one of the two segments as radius*

As we can see, in both of these aforementioned techniques new uses of the specific tool for circles have been identified by students within this activity. Specifically, students that created these strategies have given to the 'circle' tool a kind of boundary or magnification attributes in their comparison strategies. Therefore, students seem to shape the tool by loading it with potentialities for which it was not previously thought to support.

## DISCUSSION & CONCLUSIONS

Our findings implicate that, in the context of the study, students were influenced in their thinking and actions according to the capabilities and limitations of the tool (instrumentation - Trouche, 2004). It is worth-noticing that the majority of students' comparison strategies are of geometrical nature and we could relate this with the nature of Cabri, which is a software of Dynamic Geometry. At the same time the students seemed to shape the tool itself, giving it new uses for which the tool was not designed to support (tool modification, "instrumentalization" - Trouche, 2004). For example, regarding our findings about the 'circle' tool, the students gave new uses to it as a "zoom in" tool and also as a "delimiter" tool.

It is worth-noticing that students' strategies were not expressed by them in a unique way. Each student has created his/her own individual instrumented techniques regarding comparison (Vérillon & Rabardel, 1995), which were ultimately expressed through various comparison strategies, most of them unusual in the school practice.

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