Using refutational text to address the multiplication makes bigger misconception

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ABSTRACT

Understanding rational numbers is a source of difficulty with mathematics for students across ages and nationalities. This intervention study focuses on the misconception on the part of the students to think that multiplication makes the operand numbers bigger and division makes them smaller, independently of the numbers involved. In this study, a refutational text was administered to 6th grade students to help them disengage from these misconceptions. Refutational texts directly state the erroneous beliefs and immediately overturn them by presenting the alternative correct ideas in a comprehensive and persuasive way. From the 87 6th grade students that participated in the Pre/Post/Retention-test intervention study, 51 students (experimental group) received the refutational text. The results showed that the refutational text helped the students partly overcome the multiplication makes bigger misconception, and the results had a long-term effect. Not only the high, but also the low prior knowledge students were profited from the intervention. In addition, students managed to transfer the acquired knowledge about multiplication also to division items: the results showed statistically significantly less mistakes of the kind division makes smaller after the intervention. Theoretical and educational implications are discussed.

KEYWORDS

Refutational text, natural number bias, multiplication makes bigger, misconceptions, rational numbers

RÉSUMÉ

Pour des étudiants de mathématiques d'origine et d'âge variés, saisir les nombres rationnels pose des difficultés. Cette étude d'intervention s'intéresse à la fausse idée de la part des étudiants que pour n'importe quel nombre, la multiplication rend les opérandes supérieures alors que la division les réduit. Pour cette étude, un texte de réfutation (refutational text) fut distribué à des élèves de 6ème pour les aider à surmonter ce malentendu. Les textes de réfutation exposent les convictions erronées aussitôt rectifiées par les bonnes alternatives présentées d'une manière compréhensive et convaincante. Notre groupe expérimental comptant 51 parmi les 87 élèves de 6ème qui ont participé au test (pre/post/retention test) de cette étude d'intervention, a reçu ce texte de réfutation. Les résultats ont montré que le texte avait effectivement aidé les élèves à surmonter en partie le malentendu de 'l'augmentation par la multiplication', avec des effets durables. Pas seulement les élèves ayant une connaissance *préalable forte, mais aussi ceux avec une connaissance préalable faible ont profité de cette intervention. Les élèves ont par ailleurs réussi à appliquer le savoir requis sur la multiplication, à la division également ; les résultats ont montré que les erreurs du type 'réduction par division' ont statistiquement baissé. Des éventuelles implications éducatives et théoriques seront discutées par la suite.*

MOTS-CLÉS
Texte de refutation, "natural number bias", "augmentation par multiplication", fausse idée, nombres rationnels

INTRODUCTION

Understanding rational numbers is not only an important component of mathematical learning, but also a predictive factor for later mathematical achievement in algebra, geometry and statistics as well as in science and other knowledge domains (Bailey, Hoard, Nugent, & Geary, 2012). The main reason why rational numbers are an important part of the mathematics curricula worldwide is because they are necessary not only for success in school but also for later professional development (Ritchie & Bates, 2013).

Unfortunately, ample research has shown that, in a variety of ages and nationalities, individuals struggle with rational number concepts and procedures (Bailey et al., 2012; Gómez et al., 2014). Many of these difficulties can be explained by a tendency to rely on natural number knowledge when reasoning about numbers such as rational and real numbers, where natural number properties do not apply. This phenomenon is often called *whole* or *natural number bias* (hereafter NNB) (Ni & Zhou, 2005).

This study focuses on students' misconceptions about operations between rational numbers and specifically with the misconception to associate each operation with specific result size, i.e., expecting multiplication to always result in bigger numbers and division to always result in smaller numbers. Here, it is argued that this misconception stem from the natural number bias phenomenon.

The Natural Number Bias phenomenon

The interference of natural number knowledge in non-natural number contexts has been long known to mathematics teachers and mathematics education researchers (Hart, 1981) mainly as a source of systematic errors that appear in cases where rational numbers differ from natural numbers. This interference results in certain kinds of mistakes because natural and rational numbers are based on different principles and properties. Thus, prior knowledge and experience with natural numbers is most often not supportive of rational numbers which are introduced later in instruction. Actually, in certain areas of rational and real number reasoning, application of natural number rules leads to misconceptions and errors (Carpenter, Fennema, & Romberg, 1993; Gelman, 2000; Ni & Zhou, 2005; Smith, Solomon, & Carey, 2005). Such areas are the dense structure of the set of rational numbers, the ordering of the rational numbers and the operations between rational numbers.

Considering the dense structure of rational numbers, for example, it is well documented that students tend to think that, similar to natural numbers, rational and also real numbers are discrete (i.e., every number has a unique successor and there is no other natural number between two successive natural numbers). Thus, they erroneously respond that there is no number between two pseudo-successive numbers, such as 0.5 and 0.6 (Hannula, Pehkonen, Maijala, & Soro, 2006; Merenluoto & Lehtinen, 2004; Vamvakoussi & Vosniadou, 2010). In addition, considering the ordering of decimal numbers, students tend to think that – as with natural numbers – decimal numbers with more digits are larger, e.g., that 2.367 is larger than 2.6 (Nesher & Peled, 1986; Resnick et al., 1989). In a similar vein, they erroneously think that a fraction's magnitude always increases when its denominator, numerator or both increase (e.g., 249/1000 is larger than ¼), which results in mistakes in ordering fractions (DeWolf & Vosniadou, 2015; Hartnett & Gelman, 1998; Meert, Grégoire, & Noël, 2009; Stafylidou & Vosniadou, 2004).

Natural Number Bias in arithmetic operations

As mentioned above, an area to privilege the effect of NNB in rational number reasoning is the area of arithmetic operations and more specifically students' reasoning about the size of the result of the operations. Students tend to think that addition and multiplication always result in a larger number than the initial numbers involved in the operations, while subtraction and division always result in a smaller number (Bell, Swan, & Taylor, 1981; Dixon, Deets, & Bangert, 2001; Fischbein, Deri, Nello, & Marino, 1985; Greer, 1987). Well-known to teachers, this misconception has been reported in the literature since the 1920s (Thorndike, 1922 as mentioned by Krueger & Hallford, 1984). However, recently this phenomenon was re-approached from a natural number bias perspective (Christou, 2015b; Vamvakoussi, Van Dooren, & Verschaffel, 2013; Van Dooren, Lehtinen, & Verschaffel, 2015).

It is reasonable to argue that this misconception could stem from NNB, because in the context of natural number arithmetic, the result of addition or multiplication between two natural numbers (except 0 and 1) is always bigger than the operands. Likewise, the result of subtraction or division between two natural numbers is smaller than the minuend and the dividend, respectively. This is, however, not necessarily true for non-natural numbers, for which the effects of operations depend on the numbers involved. For example, multiplying or dividing with a number smaller than 1 results in a smaller or bigger outcome, respectively (e.g. 5×0.2 is smaller than 5; 5:0.2 is bigger than 5).

Empirical research has shown that this misconception systematically appear in students' mistakes when solving word problems or when dealing with operations between numbers in a strictly numerical context (Bell et al., 1981; Dixon et al., 2001; Fischbein et al., 1985; Graeber, Tirosh, & Glover, 1989; Greer, 1987, 1994; Harel & Confrey, 1994; Hart, 1981). For example, only 37% of pupils 12 to 13 years old answered correctly that 26.3×0.4 is less than 26.3 (Greer, 1987) and many students respond that 0.4×0.2=0.8 and not 0.08 (Owens, 1987), which is in-line with their belief that multiplication should bigger. In the same line, secondary students have responded that $x > x \times 2$ cannot be true (Van Hoof, Vandewalle, Verschaffel, & Van Dooren, 2015) and college students have responded that $z \times 7$ cannot be smaller than 7 (Vamvakoussi, Van Dooren, et al., 2013).

There is an ongoing discourse considering the origins of the NNB (Rips, Blomfield, & Asmuth, 2008). However, scholars who follow a developmental approach to the NNB phenomenon (Christou, 2015b; Ni & Zhou, 2005; Smith et al., 2005; Vamvakoussi, Van Dooren, et al., 2013; Van Dooren et al., 2015) agree that the origins of this bias can be traced in the construction of an initial understanding of numbers which takes place very early on in our lives through informal and formal experiences with numbers and quantities. Everyday experience with procedures such as counting sets of objects and routines such as repeating the sequence of the number words result in a construction of an initial understanding of numbers that shares the properties of natural numbers (Gelman, 2000). From this perspective, the framework theory approach to conceptual change (Vosniadou, Vamvakoussi, & Skopeliti, 2008) argues that students' primal understanding for the number concept is organized into a framework theory for number which acquires characteristics that resemble those of the mathematical concept of natural numbers. This initial number concept form students' beliefs, their interpretations and their anticipations about the properties of all numbers (Gelman, 2000;

Smith et al., 2005; Vamvakoussi, Christou, & Vosniadou, 2018; Vosniadou et al., 2008). In this context, the dominance of this initial conception for numbers may result in the NNB phenomenon that appears when reasoning with more advanced numbers such as rational and real numbers, which appear later in students' life (Vamvakoussi et al., 2018).

When it comes to operations between numbers, which is the main focus of this study, relying on this initial conception for numbers may create a strong intuition about the results that can be expected from whole-number arithmetic [e.g., *multiplication makes bigger* (Greer, 1994) or *addition makes bigger* (Dixon et al., 2001)], that is erroneously overgeneralized and applied to rational number arithmetic. Ample research has shown that students hold these misconceptions throughout their school and adult life (Vamvakoussi, Van Dooren, et al., 2013) and only mathematical experts seem to manage to disengage from them (Obersteiner, Van Hoof, Verschaffel, & Van Dooren, 2015). This shows that following the typical mathematical instruction does not result in changing such misconceptions, although it is essential for gaining a deep understanding of the number concept. This supports the necessity for special interventions designed to target the specific misconceptions as a means to help students overcome them.

Interventions that target the NNB in operations between rational numbers

Few studies have tested interventions that focus on the aforementioned misconception, considering the size of the results of the arithmetic operations. Back in the 80s, Onslow (1990) designed and implemented a game with number calculations to evoke and falsify students' beliefs that multiplication always makes bigger and division always makes smaller, which proved partly effective. In the same line, in an intervention that used clickers and a power point presentation to address the same misconception, prospective primary teachers were first asked to predict the results of given operations that were later refuted by presenting the correct result (Lim, 2011).

In a recent intervention study, erroneous examples which were used to target the *multiplication makes bigger* misconception, among others misconceptions about decimal numbers, also managed to partly reduce the effect of this misconception to students' responses (Isotani et al., 2011). An erroneous example is a description of how to solve a problem or how to answer a question, which includes at least one error. Students are challenged to find, explain and/or fix the error(s), in order to more deeply learn the domain content and develop metacognitive skills (Durkin & Rittle-Johnson, 2012; McLaren et al., 2012). Interventions with erroneous examples may reduce misconceptions, especially when they are presented to the students in a more or less explicit way (Adams et al., 2014) than in cases where they are not (Durkin & Rittle-Johnson, 2012; Isotani et al., 2011). By explicit reference, what is meant is a reference to whole number arithmetic which supports these mistakes.

In summary, the existing interventions that targeted the misconception at hand have all suggested the importance of exposing students to their mathematical misconceptions and errors before they are overturned, as a means to overcome them (Isotani et al., 2011; Lim, 2011; Onslow, 1990). The first two intervention studies (Lim, 2011; Onslow, 1990) shared the idea to create a cognitive conflict, as the necessary condition for changing the erroneous conceptions the participants hold. The conflict was created as a result of the disequilibrium that appeared between the participants' predictions that were in-line with their intuitive beliefs about the results of multiplication and division, and the actual results that were presented afterwards. Cognitive conflict describes a situation which contradicts the individuals' knowledge and beliefs (Inhelder, Sinclair, & Bovet, 1974) providing a condition for challenging them that act as a first step to also changing them. This was the main technique suggested by Fischbein and his colleagues who were some of the first to have reported these misconceptions (Bell, Fischbein, & Greer, 1984). It is not by chance that cognitive conflict situations are also proposed for situations where revision of prior knowledge needs to take place for acquiring more sophisticated knowledge, in the case of learning with conceptual change (Vosniadou, Ioannides, Dimitrakopoulou, & Papademitriou, 2001). That is because cognitive conflict condition confronts learners with the impossibility of their current conceptions, creating a cognitive disequilibrium that can lead to the discovery and development of new and more sophisticated conceptions (Forman & Cazden, 1994).

For the learners to find inconsistencies between their initial incorrect conceptions, which are based on their prior knowledge, and the more sophisticated and correct ones that are to be acquired, it is important to find ways to activate both conceptions (Van Den Broek & Kendeou, 2008). In this direction, in order to create a constructivist learning environment, students' erroneous conceptions should not only be challenged, but the correct alternative should also be presented together with a rationale on why to adopt it. This way, the students could resolve their initial erroneous conceptions, preventing the possibility of creating new misconceptions, as, this way, learners are less likely to simply add new information to their initial conceptions which could result in creating new, synthetic misconceptions (Vamvakoussi, Vosniadou, & Van Dooren, 2013; Vosniadou et al., 2008). Additionally, presenting students with a reason why the specific mistakes occur and with the possible origins of their misconceptions may provide further support in students to actively participate in the difficult and cognitively demanding process of changing them (Bell et al., 1981; Mangan, 1989; Resnick, 2006). All of the above assumptions could be applied in an intervention with the use of a refutational text.

Refutational texts are texts that aim at achieving conceptual change by explicitly stating, refuting and challenging a misconception that lies in prior knowledge (Guzzetti, Snyder, Glass, & Gamas, 1993; Hynd, 2001), while at the same time offering a more sophisticated idea to be adopted. In refutational texts, false ideas are directly stated and immediately refuted by presenting an alternative correct idea in a persuasive manner, using examples and counter examples (Lem et al., 2015; Tippett, 2010). In order for a text to be called *refutational*, it must contain two essential elements: First, it must explicitly address the incorrect conception that needs to be changed. Second, the text must explicitly state that this conception is incorrect and why, providing a new, correct one as alternative (Tippett, 2010). Previous research has shown that using refutational texts helped students overcome their misconceptions that resulted in erroneous responses and low performance rates in science (Guzzetti et al., 1993; Mason, Gava, & Boldrin, 2008; Sinatra & Broughton, 2011; Skopeliti & Vosniadou, 2006; Tippett, 2010), especially those with low prior knowledge (Diakidoy, Mouskounti, & Ioannides, 2011). Recently, refutational interventions were also used to address misconceptions in mathematics, with promising results (Christou, 2012; Lem et al., 2015).

The current study

In this study, an intervention was designed and implemented with the use of a refutational text which targeted the misconception that multiplication always makes the numbers bigger. The main hypothesis of the study was that a teaching intervention that uses a refutational text could support students to overcome their misconception to think that multiplication always increases the size of the operand numbers (Hypothesis 1). It was predicted that the students who would take the refutational text (experimental group) would outperform those who would not (control group) immediately after the intervention (post-test) and also in a delayed post-test, one month after the intervention (retention-test). It was also tested whether a refutational text would have the same impact on challenging the specific misconception both on students with high and with low level of prior knowledge. In line with previous studies using refutational text in learning sciences as mentioned above, it would be expected that students with low prior knowledge would benefit from the intervention with the refutational text (Hypothesis 2). A final research question of this study was whether the knowledge acquired from the refutational text that referred only to the *multiplication makes bigger* misconception could be transferred to division items to also address the closely related *division makes smaller* misconception.

METHOD

Participants

The participants were 87 6th grade students from public primary schools in Greece, 11 to 12 years old. The Experimental Group who received the refutational text included 51 students and the Control Group included the rest 36 students. The sample was almost equally shared to boys and girls (39 girls). Students in Greece are introduced to rational numbers in 3rd grade and from 4th grade they are systematically taught the properties of fractions and decimal numbers.

Materials

Three questionnaires were used as the Pre/Post/Retained-test, with the same design but with small differences between the tasks. These questionnaires were also used in former surveys that tested the above-mentioned misconceptions (Christou, 2015a, 2015b). The first part of each questionnaire included 28 equalities with operations between one given number and one missing number, with the result also given (e.g., 6:_=14). Students were asked to answer whether it is possible to find a number that could make the given equality true or not and to choose one of the two given alternatives: *it is possible* or *it is not possible*. The exact question was: *In the following tasks, you should respond whether you think it is possible to find a value for the missing number that would provide the given result*.

The tasks were designed to be either congruent or incongruent. Congruent tasks were in-line with students' assumed expectations about the result of multiplication and division (i.e., that multiplication always makes the initial numbers bigger and division makes them smaller). Thus, in Congruent tasks, the result was bigger than the given operands for multiplication (e.g., 6×_=11) and smaller for division (e.g., 8:_=5). On the other hand, the result for Incongruent tasks was smaller than the given operands for multiplication (e.g. 3×_= 2) and bigger for division (e.g., 3:_=8). Examples for each category of tasks are presented in Table 1. The tasks were counter-balanced across the above-mentioned categories. More specifically, five items were included in each of the four main categories. There were also incongruent tasks used that involved addition and subtraction (e.g., 5-_=7) in which the correct responses were that *it is not possible*. These tasks were included to act as buffers to avoid always having *it is possible* responses. The correct response in these tasks is that *it is possible,* because students at this age have not yet been introduced to negative numbers that falsify their initial beliefs that addition always makes bigger and subtraction always makes smaller. Eight buffer tasks were included, thus, the total number of tasks in each phase of the study were 28.

TABLE 1

Examples of items per type of operation and congruency

	Multiplication	**Division**	**Addition**	**Subtraction**
Congruent	3×_ = 8	8:_ = 5		
	6×_= 11	14: _ =5		
Incongruent	3×_= 2	6: _ = 14		
	8×_ = 3	5:_= 8		
Buffers			5+_= 2	5-_= 9
			7+_=3	5-_= 7

The second part of the questionnaire included six inequalities: An operation between two numbers was presented in one of the sides of each of them, with the operation sign missing (e.g.: 3_10>3). The students were asked to fill the missing operation by choosing one of the two given alternatives: multiplication or division. These tasks were also designed to be either congruent or incongruent. Incongruent tasks involved one natural number and one rational number smaller than 1 (e.g., 6_0.2<6); students who would respond relying on their intuition about the results of the two operations would choose the incorrect operation in these tasks. *The exact question was: What is the correct operation that would make the inequality hold? Choose one of the given alternatives*.

The refutational text

The refutational text was brief (167 words), descriptive and written in simple Greek. The refutational argumentation and all examples and counter-examples used were focused explicitly on multiplication. The text started by stating that it is very common to think that multiplication always results in numbers that are larger than the operand numbers and this is often supported by examples of multiplication between natural numbers – such examples were also given (i.e., 2×3=6). This statement was immediately refuted by stating that although this is true for multiplication between natural numbers, it is not always true when non-natural numbers are involved. The argument was that, actually, when numbers larger than 1 are multiplied, multiplication makes both operand numbers bigger. However, when the numbers are smaller than 1, the result is smaller than the operand numbers. Two examples were given where one or both operand numbers were smaller than 1 (8×1/2=4, 0.7×0.8=0.56) and one example with one of the operand numbers being an improper fraction (i.e., which size is bigger than one: 3×4/3=12/3=4). The examples were accompanied with comments that drew the attention on comparing the size of the operands with the results.

At the end of the refutational text, four comprehension questions were included, in order to force students to reflect on the information presented and thus increase the profit of the refutational text (Tippett, 2010). The questions were: a) If two numbers are multiplied to each other, is the result bigger or smaller than the operand numbers? b) When is it true that the multiplication between two numbers results in a number bigger than the first two? c) Can you give an example where multiplication can make smaller? d) How would you respond to the above question if you had not read the text? Did you change your initial viewpoint? In what way?

Procedure

The students completed the tests in their classroom during their mathematics course with the presence of their teacher and the researcher. First, the Pre-test was given to the students of the Experimental Group, followed by the refutational text and the Post-test. The Control Group did not receive the refutational text or any other instruction about the size of the results of the operations between rational numbers.

When students were given the questionnaires, they were told that they should be aware that there is only one correct answer for each question. The questionnaires also contained some simple instructions for completing them. For the first part of the questionnaire, students were told to choose one of the two given alternatives that best represented their opinion. They were also explicitly told that they could think with any kind of number they know. The second part of the questionnaire included a note with the meanings of the symbols > and <.

Students were asked to read the refutational text individually and to answer the four questions that followed. Clarification questions were answered. Adequate time was provided to complete each of the given questionnaires and to read the refutational text. Specifically, in the experimental group, Pre-test lasted 20’, reading the refutational text and answering the comprehension question lasted 45' and the Post-test lasted 20'; the Retention-test lasted 20' and took place one month after the intervention.

RESULTS

Performance in the equalities

Participants' responses were scored on a right/wrong basis and mean scores were calculated for each category of tasks. Table 2 presents students' mean performance in the incongruent equalities that involved multiplication, for each of the three phases of the experiment. Starting with the first part of the questionnaires that included equalities between given and missing numbers, students' mean performance in the incongruent multiplication items showed that the experimental group performed slightly better than the control group in the Pre-test; however, a Levene's test verified the equality of variances in the sample (homogeneity of variance) ($p>.05$), indicating that the two groups were comparable. In-line with the main hypothesis of the study, in the Post-test, immediately after the intervention with the refutational text, the experimental group responded statistically significantly higher in the incongruent equality tasks, compared to the Pre-test $t(50)=2.006$, $p=0.05$. Additionally, the students from the experimental group also performed statistically significantly higher in the Retention-test that was administered one month after the intervention, compared with the Pre-Test $t(49)=3{,}758$, $p<.001$ (Prediction1).

On the other hand, the control group did not show significant differences in their performance on the incongruent equalities that involved multiplication, either between the Pre-test and the Post-test $t(35)=.924$, $p=.362$ or between the Pre-Test and the Retention-test $t(35)=1.528$, $p=.136$.

TABLE 2

Mean performance in incongruent equality tasks in multiplication

		Mean	Std. error	Min	Max
Experimental Group	Pre-test	.09	.03	0	1
	Post-test	.16	.04	0	1
	Retention-test	.27	.05	0	1
Control Group	Pre-test	.06	.02	0	.4
	Post-test	.10	.04	0	1
	Retention-test	.13	.04	0	1

Testing knowledge transfer

In order to test possible knowledge transfer from multiplication to division, the same analysis was followed for the division tasks, starting from students' performance in the equalities. Table 3 presents the mean performance on incongruent division tasks in the equalities that were included in the first part of the questionnaire. Students from the control group appeared to perform higher than students from the experimental group; however, again, a Levene's test verified the equality of variances in the sample ($p>.05$), indicating that the two groups were comparable. In the Post-test, immediately following the intervention with the refutational text, the experimental group responded higher in the incongruent equality tasks in division, compared to the Pre-test; however, this difference was close but not statistically significant $t(50)=1.863$, $p=0.068$. Nevertheless, the experimental group performed statistically significantly higher in the Retention-test that was administered one month after the intervention, compared with the Pre-Test, $t(49)=3{,}321$, $p<.05$.

The control group did not show statistically significant differences in their performance on the incongruent division tasks in equalities either between the Pre-test and the Post-test *t*(35)=.595, p=.556 or between the Pre-test and the Retention-test *t*(35)=.095, p=.925.

Performance in the inequalities

Quite similar were the results in the second part of the questionnaire that included inequalities and students were asked to choose the operation that would make the inequality hold. Students' responses in these tasks were combined between multiplication and division in order to raise the statistical power of the analysis that was followed, since the items used were not as many as in the first part of the questionnaires that included equalities. In the Pre-test, the students from the experimental group appeared to perform higher (M=.34, SE=.05) than the students from the control group (M=.28, SE=.06), however, a Levene's test verified the equality of variances in the sample (p>.05), which again shows that the groups are comparable. Students from the experimental group performed slightly better in the incongruent inequality tasks in the Post-test immediately after the intervention (M=.35, SE=.05) compared with the Pre-test before the intervention, but this difference was not statistically significant *t*(50)=.104, p=.918. However, the students from this group performed statistically significantly higher in the Retention-test (M=.48, SE=.05), one month after the intervention, than in the Pre-test *t*(49)=2,007, p=.05.

In addition, students from the control group did not show any statistically significant difference in their performance on the incongruent inequalities before the intervention compared with either immediately after the intervention (M=.27, SE=.07) *t*(35)=.291, p=.773, or one month later (M=.28, SE=.06), *t*(35)=.192, p=.849.

TABLE 3

Mean performance in incongruent equalities tasks in division

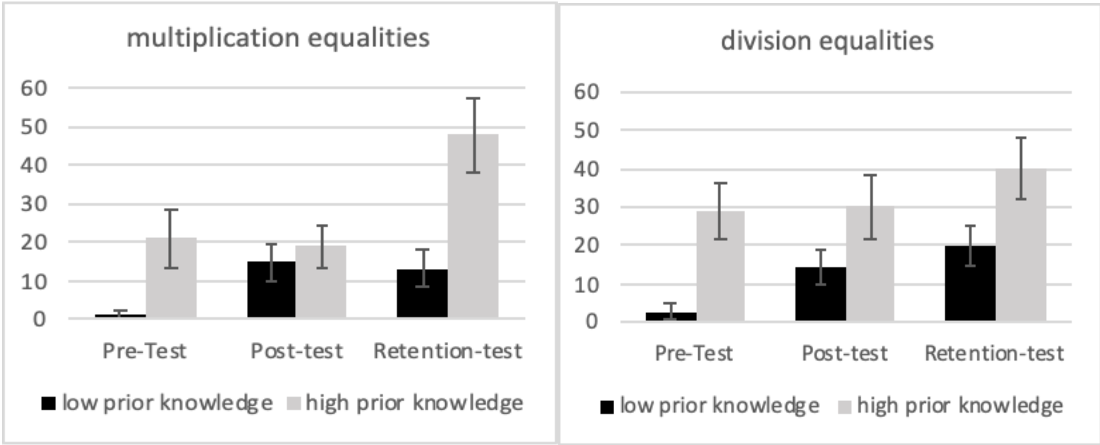
		Mean	Std. error	Min	Max
Experimental Group	Pre-test	.13	.04	0	1
	Post-test	.20	.04	0	1
	Retention-test	.28	.04	0	1
Control Group	Pre-test	.19	.05	0	1
	Post-test	.20	.06	0	1
	Retention-test	.18	.05	0	1

Testing for the effect on low vs high prior knowledge students

To test whether the intervention had a different effect on students with low versus high prior knowledge, an additional analysis was conducted. First, students from the experimental group were classified based on a median split on pretest score, with 31 students classified as *low prior knowledge* (i.e., with Pre-test score less than 16 points: M=14.12, SD=1.78, min/max=11/16) and 20 students classified as *high prior knowledge* (i.e., with Pre-test score more than 17 points: M=20.1, SD=4.02, min/max=17/34). The performance of each of these groups in the three phases of the experiment are presented in Figure 2.

The group with high prior knowledge students performed statistically significantly better on the incongruent multiplication equalities only one month after the intervention with the refutational text *t*(19)=2.896, p=.009, but not in the post-test, immediately after the intervention *t*(19)=.400, p=.694. This shows that the refutational text had long term effects to those with high prior knowledge. Considering their ability to transfer their acquired knowledge to the division tasks, the group of high prior knowledge did not show any change in its performance in the incongruent division equalities immediately after the intervention compared with its performance before [*t*(19)=.131, p=.897], or one month after the intervention compared with its performance before the intervention [*t*(19)=1.351, p=.192].

FIGURE 1



Accuracy rates by Test and by Group for multiplication and division equalities

In-line with the second hypothesis (Hypothesis 2) the group of low prior knowledge performed statistically significantly higher on the incongruent multiplication equalities immediately after the intervention compared with its performance before it *t*(30)=2.742, p=.010 and the results were retained one month after the intervention with the refutational text *t*(29)=2.523, p=.017. Considering its ability to transfer their knowledge to the division tasks, the low prior knowledge group of participants showed statistically significant higher performance on the incongruent division equalities immediately after the intervention compared with its performance before [*t*(30)=2.683, p=.012], and these results were retained one month after the intervention [*t*(29)=3.372, p=.002].

DISCUSSION

The present study aimed to test the efficacy of an intervention, using a refutational text that targeted the misconception on the part of students to associate multiplication and division with a specific size of result, independently of the numbers involved. This misconception is often characterized as the *multiplication makes bigger* misconception (Bell et al., 1984; Fischbein et al., 1985; Greer, 1987), because of students' tendency to think that multiplication always provides bigger results than the initial operand numbers. A closely related misconception is to also think that division *always makes numbers smaller*.

The refutational text that was administered to an experimental group of primary school students initially stated the specific misconception at hand and, immediately after, it refuted it by presenting examples that showed that multiplication can result in numbers smaller than the operand numbers as well. Also, it was specifically mentioned that the tendency to anticipate bigger numbers as results of multiplication is due to previous experience with natural numbers, where multiplication indeed makes numbers bigger. Students' responses in the four comprehension questions that accompanied the text showed that the students read and understood the text.

The results showed that the refutational text was successful in helping students partly overcome their misconception that multiplication always makes numbers bigger. The students from the experimental group scored statistically significantly higher in the incongruent tasks in multiplication after the intervention, and these learning gains were retained one month later. The finding that the control group did not show any statistically significant change in their responses in the following tests allows to interpret these results as a learning gain acquired from the refutational text. These results further support previous findings considering the effectiveness of using refutational texts to address misconceptions in science and mathematics (Diakidoy et al., 2011; Guzzetti et al., 1993; Hynd, 2001; Lem et al., 2015; Skopeliti & Vosniadou, 2006).

The results of the current study also showed that the refutational text helped students gain deeper knowledge considering the size of the results of operations and this appeared in students' higher performing rates in the division equality items that were included in the tests. More specifically, students managed to increase the correct response rates in the incongruent division equalities, showing traces of knowledge transfer from multiplication to division. It should be noted here that the refutational text that was administered to the students referred only to the misconception conserving the multiplication and there was no reference to division. Also, the examples and counter examples used included explicitly multiplication and were provided in the context of equalities. Thus, higher performing rates in division tasks for the experimental group in the Pest and the Retention-test may be interpreted to result from transferring the knowledge that was acquired from multiplication to division, indicating deeper knowledge gains (Perry, 1991; Rittle-Johnson & Alibali, 1999).

In the same line were also the results from the second part of the questionnaires that included inequalities. In order to respond correctly in these tasks, someone should think more carefully and apply the properties of the operations that were raised in the refutational text. The statistically significant higher performing rates in these tasks after the intervention only for the experimental group may also be interpreted as a learning profit from reading the refutational text, which may help students acquire deeper knowledge of the properties of operations rather than staying on the procedural knowledge level (Rittle-Johnson & Alibali, 1999).

Finally, in line with the second hypothesis of the study, the results showed that not only students with high prior-knowledge, but also the low prior knowledge students profited by the specific intervention. Actually, interestingly, the group of low prior knowledge students appeared to have profited more than those with high-prior knowledge. Specifically, low prior knowledge students' learning gains appeared immediately, as well as one month after the intervention, while the group of students with high prior knowledge appear to have profited only at the long run. Also, the group of students with low prior knowledge appeared to transfer their learning gains from multiplication to division much more than the students from the high prior knowledge group. Future studies using qualitative methods with clinical interviews could provide more insights for this discrepancy.

This study has several theoretical and educational implications. The findings suggest that the refutational text may help students partly disengage from their misconceptions in mathematics and implies that they could be included more often in the mathematical textbooks together with the expository text already used. However, the results showed that the misconception was far from being eradicated. Students' mean performance in the incongruent tasks of the Post-test and the retention-test continued to be very low and, thus, the positive effects of refutational text are considered as moderate. This shows that the *multiplication makes bigger* and the *division makes smaller* misconceptions are difficult to overcome, supporting previous findings in the field, and showing that they are not just mistakes that students do, but they stem from certain beliefs that lie in deep rooted knowledge structures that needs revision (Christou, 2015b; Ni & Zhou, 2005; Obersteiner et al., 2015; Vamvakoussi, Van Dooren, et al., 2013; Van Hoof et al., 2015). For students to fully disengage from misconceptions like those that stem from the natural number bias, they would need to develop a number concept that goes beyond the natural numbers and is closer to the mathematical concept of number, such as incorporating the properties of the rational and the real numbers (Smith et al., 2005). This entails learning with conceptual change, because, in this process, the students will need to fundamentally reorganize their initial framework for numbers which is organized around the symbols and properties of natural numbers (Vamvakoussi, Vosniadou, et al., 2013; Vosniadou et al., 2008). Interventions with refutational texts such as the one implemented in the present study may assist students in this process which is expected to be difficult and time consuming.

Despite certain drawbacks, the results of this study are very important mainly because they show that a short intervention with a refutational text can make a difference in the process of challenging the misconceptions at hand. This could suggest that such texts can be included in more powerful learning environments that are designed to teach the properties of rational numbers in ways that would provide bigger help for students to exceed their difficulties and misconceptions. These learning environments should include meaningful representations about the operations between rational numbers. Teaching other models for multiplication than the repeated addition model which supports the *multiplication makes bigger* misconception, such as the *area model*, the *fractional part of a quantity* model or the *scaling* model for multiplication, could be beneficial for the students in the long run (Prediger, 2008). The same holds for the *partitive model* for division that is associated with the *division makes smaller* misconception, which could be presented together with other models such as the *quotative* or *measurement model* of division that are more susceptible to the properties of operations between rational numbers (Bell et al., 1984; Fischbein et al., 1985; Lim, 2011).

Still, refutational argumentation should be used with caution. Together with other persuasion techniques, refutational texts have been criticized as authoritative, manipulative and male dominated (Anders & Commeyras, 1998 for a thorough discussion see Hynd, 2001). From our perspective, believing that, by presenting the mathematically correct conception in a refutational argumentation format, students are forced to unquestionably accept it, as it underestimates their ability to think critically and to self-construct own beliefs and conceptions. However, taking these critiques under consideration, using that refutational argumentation methodology to start a class discussion about the targeted misconceptions could prove an even more profitable and more inclusive teaching technique than when just presenting to the students the arguments against their incorrect beliefs and ideas (Inagaki, Hatano, & Morita, 1998). Of course, all these plausible teaching alternatives need to be empirically tested.

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