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Effects of Concrete-Representational-Abstract instruction on fractions among low-achieving sixth-grade students

CHARALAMPOS LEMONIDIS[1], DIMITRIS ANASTASIOU[2], THALIA ILIADOU[1]

[1]*Department of Primary Education*
University of Western Macedonia
Greece
xlemon@uowm.gr
thaliailiadou25@yahoo.gr

[2]*Department of Counseling, Quantitative*
Methods and Special Education
Southern Illinois University Carbondale
USA
anastasiou@siu.edu

ABSTRACT

The purpose of this study is to examine the effects of an explicit Concrete-Representational-Abstract (CRA) approach that emphasised conceptual knowledge of fractions on low-achieving sixth-grade students. Participants were 34 sixth-grade students, of which 10 showed low achievement in fractions (LAF) and 24 exhibited typical achievement in fractions (TAF). The explicit CRA intervention resulted in equally large effect sizes for LAF and TAF students. In addition, the large gap in fractions performance between LAF and TAF students, observed at the pre-test time, was significantly bridged at the post-test time. The effect size of the gain in the conceptual knowledge of fractions was large among LAF students. Results are discussed in terms of the CRA instructional effectiveness in fractions. Finally, instruction efficiency issues within diverse general education classrooms are considered.

KEYWORDS

Fractions, concrete-representational-abstract approach, conceptual knowledge, low-achieving students, CRA effectiveness, instruction efficiency

RÉSUMÉ

*L'objectif de cette étude c'est d'examiner les effets d'une approche explicite Concrète-Représentationnelle-*Abstraite *(CRA) qui mettrait l'accent sur la connaissance conceptuelle des fractions chez les élèves de la 6ème avec des résultats peu satisfaisants. Les participants étaient 34 élèves de la 6ème, dont 10 ont montré des résultats peu satisfaisants en fractions (LAF) et 24 ont montré une connaissance typique en fractions (TAF). L'intervention explicite (CRA) résulté en tout aussi grand effect sizes pour les élèves LAF et TAF. De plus, l'écart important dans la performance des fractions entre les élèves LAF et TAF, observé au moment du pré-test, a été significativement réduit au moment du post-test. L'effect size était important dans la connaissance conceptuelle des fractions chez les élèves LAF. Les résultats sont discutés en termes d'efficacité* *pédagogique CRA en fractions. Enfin, nous examinons les problèmes d'efficacité didactique dans diverses classes d'enseignement général.*

MOTS-CLÉS
Fractions, approche concrète-représentationnelle-abstraite, connaissances conceptuelles, élèves peu performants, CRA efficacité, efficacité de l'instruction

INTRODUCTION

Understanding fractions can support students' learning of proportions, ratios, decimals, percentages, and rational numbers, as well as their learning of advanced mathematics such as algebra and geometry. Competence with fractions is important for solving problems in science, technology, and engineering, and for dealing with everyday activities (Hoyles, Noss, & Pozzi, 2001; Jordan et al., 2013; National Mathematics Advisory Panel, 2008; Siegler et al., 2012). Developing proficiency with fractions is important for physical, biological, and social sciences. Fractions are also important for a wide range of middle-income occupations, such as nursing, carpentry, and auto mechanics (Hoyles et al., 2001; Sformo, 2008).

Several studies have shown that children and adults have difficulty understanding fractions (e.g. Aksu, 1997; Lemonidis, Tsakiridou, & Meliopoulou, 2018; Lortie-Forgues & Siegler, 2015; Mack, 1995; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2010). Learning fractions is especially difficult for students with mathematics difficulties (MDs). Students with MDs have serious difficulties in ordering fractions from the least to the greatest and in identifying equivalent fractions (Grobecker, 2000; Hecht & Vagi, 2010; Mazzocco & Devlin, 2008). A deficit in the conceptual understanding of fractions, such as differences in fraction magnitude knowledge, can restrict students' ability to apply routine computational procedures involving fractions (Siegler et al., 2010; Siegler & Pyke, 2013). Bailey and colleagues (2015) found that Chinese middle-school students had a greater knowledge of fraction concepts and procedures than did their U.S. peers. In China, even children with MDs who perform in the bottom third showed reasonably good knowledge of both fraction magnitudes and fraction arithmetic. Tian and Siegler (2017) suggested that high-quality teaching and substantial practice could allow children with MDs to acquire reasonably good fraction knowledge. Following Tian and Siegler's suggestion, the curriculum of this interventional study includes both procedural knowledge tasks relevant to the operations of fractions (e.g. subtracting mixed fractions by whole number, conversion of mixed fractions) and conceptual knowledge tasks relevant to the magnitude of fractions on number line, equivalent fractions, and fractions comparison.

A rationale for a curricular approach to fractions instruction

Greek students receive traditional fractions instruction based on a part–whole approach and algorithmic rules without the use of number line. On the other hand, instruction in fraction magnitudes using number line representations has been a common feature in most experimental interventions (Fuchs et al., 2013, 2014, 2016, 2017; Lemonidis, 2017; Saxe, Diakow, & Gearhart, 2012; Schneider, Grabner, & Paetsch, 2009). Saxe, Diakow, and Gearhart (2012) proposed a curriculum unit for fourth- and fifth-graders that utilized number line as the principal representation in order to enhance fraction understanding. In their study, LAF students (defined as those in the bottom third of the distribution) performed similarly to median achievers (those in the middle third of the distribution) when using such a curricular approach.

This magnitude interpretation of fractions using a number line was included in our intervention programme. Such an approach has several advantages over the traditional Greek method. First, the magnitude representation of fractions on a number line helps in understanding a fraction as a number. Second, improper fractions are introduced more easily on a number line, and the continuity of improper fractions with other numbers is apparent. Third, the continuity of a number line can show the continuity between numbers and can make clear that there is an infinite number of fractions between any two fractions; there is not just one fraction number after any other given fraction. Fourth, a number line is a conventional mode of representing operations with fractions, such as the addition and subtraction of fractions with like denominators and the multiplication of a natural number with a fraction.

Explicit, systematic, and direct instruction in fractions

Explicit and systematic fraction instruction that includes manipulatives and visuals has been found to be effective for both students with mathematics disabilities and those at risk for developing mathematics disabilities across a wide range of grades (Bottge & Hasselbring, 1993; Bottge et al., 2014; Flores & Kaylor, 2007; Hunt, 2014; Scarlato & Burr, 2002). Misquitta's (2011) review of the literature revealed that three types of intervention were found to be effective in teaching fractions: graduated sequence [e.g., concrete-representational-abstract), strategy instruction (e.g., cue card strategy, LAP fractions strategy: "(L)ook at the denominator and sign to determine if fractions were like or unlike, and what operation was required; (A)sk themselves if the denominators would divide evenly into each other; and (P)ick a fraction type" (Misquitta, 2011, p. 114)], and direct instruction.

Explicit instruction, wherein nothing is left to chance, was necessary to improve students' performance in fractions (Bouck & Sprick, 2019). In their research synthesis, Shin and Bryant (2015) found that 10 of 17 studies that used concrete and visual representations in combination with explicit and systematic instruction yielded highly positive outcomes on fraction concepts and skills. An explicit instruction typically follows an instructional sequence: *use of an advance organizer*, *teacher modelling skills*, providing *gradual release* with prompts and cues while the student works to solve fraction problems, and having the student *independently* solve problems (Bouck & Sprick, 2019; Doabler & Fien, 2013).

For the purpose of this study, *direct instruction* is defined as a skills-oriented approach and teacher-directed approach using presentations or demonstrations of the material, rather than exploratory methods of learning. Following Huitt, Monetti and Hummel's (2009) general model of *direct instruction*, we followed a model containing five instructional components: (a) presentation of the instructional content, (b) modelling and guiding, (c) semi-independent and independent practice, (d) assessment and evaluation, and (e) monitoring and feedback.

Finally, *systematic instruction* adheres to a fixed plan by using a specific order for introducing new information. CRA is considered a systematic instructional approach that follows an organized and graduated sequence (Pullen & Hallahan, 2015). Our intervention programme was based on a systematic instruction following a Concrete-Representational-Abstract (CRA) approach with the aforementioned explicit and direct instructional elements.

The CRA approach is also referred to as concrete-semiconcrete-abstract framework (Flores, Burton, & Hinton, 2018). In the CRA approach, the term *concrete* refers to materials, such as commercially available fraction circles, fraction tiles and other manipulative devices (Butler et al., 2003; Bouck & Sprick, 2019). The term *representation* refers to pictorial techniques, such as drawings. Visual representations are crucial to understanding basic fraction concepts (e.g., equivalent parts of a whole) (Butler et al., 2003). The *abstract* part of the approach refers to symbolic numerals, that is, fractions and whole numbers without pictorial representation.

CRA interventional research in fractions

CRA studies have shown that LAF students can develop conceptual understanding of mathematics across several mathematics content areas, including fractions (e.g. Butler et al., 2003; Jordan, Miller, & Mercer, 1999; Sun, Peishi, & Craig 2015). Jordan and colleagues (1999) found that students who were taught fraction concepts and skills following a CRA sequence performed significantly better than students who were taught on the basis of a traditional textbook curriculum. A CRA sequence was effective in helping students to gain skills in fraction identification, comparison, equivalence, and computation. Butler and colleagues (2003) compared the effects of a CRA sequence with those of a representational-abstract (RA) sequence (pictorial and numerical quantities alone without manipulatives) on equivalent fraction concepts and procedures among 6th- to 8th-grade students with mathematics disabilities. The CRA group used concrete manipulative devices for the first three lessons, while the RA group used representational drawings. Students in both treatment groups improved their overall understanding of fraction equivalents. Although the pattern of post-treatment differences was not statistically significant, the CRA students tended to have higher average scores than did the RA students. Sun, Peishi and Craig (2015) examined the effects of fraction-word problem-solving instruction involving the explicit teaching of the CRA sequence with culturally relevant examples for three low-performing fourth-grade Asian immigrants who were English Language learners. The three participants reached grade-level mastery on math word problems.

The research questions of this study are as follows:

(1) What are the overall effects of the explicit CRA instruction on the fractional knowledge and skills of the LAF students?
(2) What are the specific effects of the explicit CRA instruction on problem solving, procedural knowledge, and conceptual knowledge of fractions among the LAF students?
(3) What are the effects of the explicit CRA instruction on pictorial representation of fractions among the LAF students?

METHOD

Participants

Participants were 34 sixth-grade students from two general education classrooms of two different schools in a provincial town in Western Greece. Students were formed into two groups according to their performance in a pre-test and their evaluation by teachers. The classroom teachers indicated the students with typical performance in maths, and those with difficulties in math. The teachers' evaluations were consistent with the students' test results. Based on pre-test performance pre-test and evaluation by teachers, children with LAF were below the 30th percentile. Jordan, Hanich and Kaplan (2003) followed similar procedures for the sample selection of students with low achievement in mathematics. The low achievement in fractions (LAF) group consisted of 10 students (7 males and 3 females), and the typical achievement in fractions (TAF) group consisted of 24 students (9 males and 15 females).

Measures

Both the pre-test and post-test consisted of 10 problems. Eight of these problems can be grouped into three fraction areas: problem solving, procedural tasks, and conceptual tasks. Two other problems, the part–whole area model and the equipartitioning figures, were outside of this classification. The three target fraction tasks are described below in detail.

Problem solving. This fraction area consisted of three problems with fractions as follows:

Problem 1 (Pr1): 'Each of four children ate 3/5 of pie. How many pies did they eat altogether?' In addition, students were asked here to 'make a drawing to represent the operation'. An answer was scored as correct when students either gave an accurate algorithmic answer to 4 x 3/5=12/5 using their procedural knowledge or conceptually represented the solution using drawings in an accurate way.

Problem 2 (Pr2): 'Laura and Emilia tried to finish their mathematics exercises worksheet before the beginning of their favourite cartoon programme on TV. Laura completed 1/2 of the exercises and Emilia 4/5 of the exercises. Which of the two girls has completed more exercises in the worksheet? (A) Laura or (B) Emilia. Why? Please, justify your answer.' In this problem, a credit was given only when students could give correct answers with justification.

Problem 3 (Pr3): 'Nick ate the 3/12 of a cake, George ate 4/12, and Irene ate 2/12 of the cake. How many parts of the cake did the three children eat altogether? How much of the cake was left?' An answer was scored as correct when students either gave an accurate algorithmic answer or represented the solution using drawings in an accurate way.

We consider that problems with fractions represent an important instructional objective, as fraction knowledge is significant to advanced mathematics (Every Child a Chance Trust, 2009; Gross, 2009; Murnane et al., 2001; Parsons & Bynner, 1997).

Procedural knowledge tasks. This fraction area consisted of two tasks. P1: Students were asked to make a subtraction of a mixed fraction from a whole number (9-6 ¼); and P2: Students were asked to convert a mixed fraction (5 ¼) into an improper fraction.

Conceptual knowledge tasks. This fraction area consisted of three tasks. C1: Students were asked to position 2/3 on a number line; C2: Students were asked to find equivalent fractions ('Tell me two equivalent fractions (of equal value) with 2/3'); and C3: Students were asked to make fraction comparisons ('Make a comparison between 1/7 and 1/8. Which has greater value and which less? Please, justify your answer.')

Rittle-Johnson, Siegler and Alibali (2001) defined *conceptual knowledge* as 'implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain. This knowledge is flexible and not tied to specific problem types and is therefore generalizable' (pp. 346-347). Conceptual knowledge is distinguished from *procedural knowledge*, which refers to 'the ability to execute action sequences to solve problems. This type of knowledge is tied to specific problem types and therefore is not widely generalizable' (Rittle-Johnson et al., 2001, p. 346).

Intervention

Content

The intervention was based on the Common Core State Standards for Mathematics (CCSS-M 2010) framework adapted to the Greek educational system and adjusted for number sense of fractions (Lemonidis, 2015). Specifically, the following fraction topics were taught: (a) fraction placement on the number line; (b) fraction concepts, based on unitary fractions; (c) understanding that fraction value can be greater or less than 1; (d) equivalent fractions; (e) fraction comparison using the number line; (f) addition and subtraction of fractions with like denominators using unitary fractions; (g) conversion of mixed fractions into improper fractions and improper fractions into mixed fractions; (h) addition and subtraction of mixed fractions; and (i) multiplication of fractions by whole numbers.

Teaching sequence

Each lesson format had a similar structure to that used in the study by Butler and colleagues (2003), taking Huitt and colleagues' (2009) suggestions into consideration. Specifically, each instructional session included five stages of explicit instruction in each of the three modes of representation – concrete, representational, and abstract – as follows: (1) *Advance organizer*. The teacher introduced students to the lesson objectives, making connections with prior concepts and skills; (2) *Modelling*. The teacher modelled how to solve a fraction problem, using *think-alouds,* that is, verbalising what they thought while performing a fraction task using concrete materials (e.g. blocks, number line), representational techniques (mainly drawings), and abstract methods; (3) *Guided practice*. The teacher guided students to fraction activities. Through a gradual release intermediate phase, students gradually took the lead in completing tasks using a concrete to representational to abstract sequence with immediate feedback from the teacher. Encouragement of the concrete to representational to abstract sequence through cue cards was frequent; (4) *Independent practice*. Students performed tasks independently (see Butler et al., 2003); (5) *Assessment and progress monitoring*. At the end of each session, students completed a curriculum-based assessment. Thus, teachers could monitor students' progress, and/or students could self-monitor their own progress.

Delivery and training

In total, 21 instructional sessions took place, each lasting 45 minutes and within a one-month period. Before the intervention, two teachers were trained in educational materials and teaching methods. The educational material was provided by the researchers to the teachers. The intervention was the same for all students in both groups. The two trained teachers who implemented the intervention were interviewed afterward to explore their views on the intervention.

Fidelity of implementation

Scripted lessons, manipulatives, worksheets for guided and independent practice, and curriculum-based assessments were provided by the researchers. The completed curriculum-based assessment protocols were at the disposition of the researchers. In addition, throughout the programme, teachers maintained an observation protocol, recording their activities and students' responses, and made comments while they were performing the activities. One of the researchers conducted 12 school field visits to ensure that the teachers implemented the instructional programme properly. Phone calls were also frequent. Adherence to the elements of the implemented intervention was 90%.

Procedure

Before the intervention, students were assessed with a pre-test, which was intended to evaluate their knowledge and skills in fractions and to identify LAF students. The pre-test consisted of 10 fraction problems; each correct answer was scored with 1 and each incorrect answer scored with 0. The average performance of the LAF group was in the bottom 30% of the distribution, a criterion that has been applied by other studies (e.g. Gersten, Jordan, & Flojo, 2005). In addition, teachers' suggestions about LAF and TAF students led to the identification of the same students. The post-intervention test consisted of the same 10 fraction problems, identical to those in the pre-test. All students were tested with the post-test, three weeks after the completion of the instructional intervention.

RESULTS

Table 1 shows the descriptive statistics for problem-solving, procedural knowledge, and conceptual knowledge tasks in the pre-test and post-test.

TABLE 1

Mean and Standard Deviation of Correct Answers in Pertest and Post-test among LAF and TAF students*

		Pre-test				**Post-test**			
Fraction area	**Task**	LAF, *N*=10		TAF, *N*=24		LAF, *N*=10		TAF, *N*=24	
		M	***s***	***M***	***s***	***M***	***s***	***M***	***s***
Problem solving	**Pr1**	0.10	0.32	0.54	0.51	0.80	0.42	0.92	0.28
	Pr2	0.00	0.00	0.71	0.46	0.80	0.42	0.88	0.34
	Pr3	0.00	0.00	0.50	0.51	0.90	0.32	0.88	0.34
Procedural knowledge	**P1**	0.20	0.42	0.71	0.46	0.70	0.48	0.83	0.38
	P2	0.20	0.42	0.88	0.34	0.70	0.48	0.92	0.28
Conceptual knowledge	**C1**	0.00	0.00	0.21	0.42	0.80	0.42	0.96	0.20
	C2	0.40	0.52	0.88	0.34	1.00	0.00	1.00	0.00
	C3	0.30	0.48	0.71	0.46	0.90	0.32	0.88	0.34

*Note: The mean values refer to percentages of students who solved a problem correctly.

Pre-test performance

Overall, problem solving with fractions and positioning fractions on a number line were the most difficult tasks for LAF and TAF students at the pre-test time. It is noteworthy that the conceptual task of positioning 2/3 on a number line (C1) was the most challenging task for both groups. None of the LAF students and only five (20.8%) of TAF students responded accurately. It should be noted that students are not taught to use a number line in fractions, since this is not part of the Greek common-core curriculum.

The overall effects of the explicit CRA instruction on fractions

Table 2 shows the medians, means, standard deviations, and effect sizes of the overall fraction performance for both LAF and TAF student groups before and after the intervention. The effect size r for both Mahn-Whitney U test and Wilcoxon signed-rank test was calculated by dividing the standardized z-score values by the square root of the number of observations over the two time points (Rosenthal, 1994).

At the pre-test, TAF students statistically significantly outperformed LAF students (U = 240, z = 4.583, $p < 0.001$, r = .56). However, at the post-test, the initial gap in overall fractions performance between the two groups was bridged, since their difference was no longer statistically significant (U = 136, z = .659, p = 0.51), and the effect size was negligible (r = .08).

The intervention resulted in improving the overall fraction performance for both groups. The LAF students showed an overall gain of +5.90 points at the post-test. The Wilcoxon signed-rank test showed that this improvement was statistically significant (T = 55, z = 2.840, $p < 0.01$) with a large effect size (r = 0.64), per Cohen's (1988) convention for the effect sizes. The TAF students also significantly increased (+2.50 overall points) their fraction performance at the post-test (T = 253, z = 4.143, $p < 0.001$), with a large effect size (r = .60) (see details in Table 2).

TABLE 2

Overall Performance of Two Groups at Pre-test and Post-test

	LAF Group			TAF Group			Total		
	Mdn	***M***	***s***	***Mdn***	***M***	***s***	***Mdn***	***M***	***s***
Pre-test	3.0	2.60	1.35	6.5	6.71	1.63	5.00	5.50	2.44
Post-test	9.5	8.50	2.07	10.0	9.21	1.02	10.00	9.00	1.44
Average difference		+5.90			+2.50			+4.50	
Effect size *r*		**.64**			**.60**			**.60**	

Note. Max score: 10

The specific effects of the explicit CRA instruction per fraction domain

Table 1 indicates the details of the performance change of the two groups between pre-test and post-test time. The *problem-solving* (Pr1, Pr2, and Pr3) tasks were the most challenging in the pre-test for both groups; at the post-test, students exhibited a significant improvement in these tasks (Table 1). The LAF students, who scored from 0 to 10% at the pre-test, reached 80% to 90% success at the post-test. Using the Wilcoxon signed-rank test, we found that this gain was significant (T = 55, $p <$ 0.01, r = .42). The TAF students also showed a significant gain (T = 153, $p < 0.001$, r = .55).

In the *procedural tasks*, students had to execute the subtraction 9 − 6¼ (P1) and to convert 5¼ to an improper fraction (P2). LAF students, with a 20% success rate in both tasks at the pre-test, reached 70% at the post-test. Using the Wilcoxon signed-rank test, we found that this gain was significant (T = 21, $p < 0.05$, r = .51). In addition, the TAF students showed significant gain significant with a smaller effect size (T = 10, $p < 0.05$, r = .29).

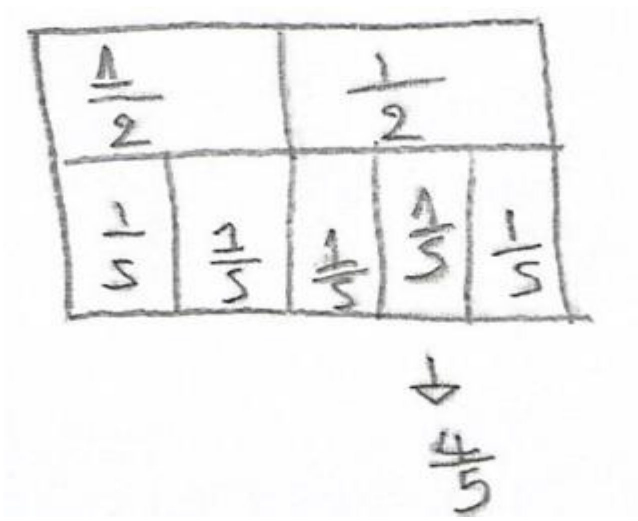
Regarding the *conceptual tasks*, placing the fraction 2/3 on a number line (C1) was one of the most difficult tasks of the pre-test for both groups of students; in the post-test, both groups made noticeable gains: in the LAF group, from 0% to 80%, and in the TAF group, from 20.8% to 95.8%. The Wilcoxon signed-rank test showed that this improvement in the conceptual tasks was significant, with large effect sizes for both the LAF group (T = 55, $p < 0.01$, r = .64) and TAF group (T = 55, $p < 0.001$, r = .56). This massive improvement by the students is probably because the number line was the basic tool of intervention instruction; thus, there was a direct alignment of instruction with post-testing.

Qualitative analysis of the use of procedural strategies and conceptual knowledge

At the pre-test, most TAF students applied rules and procedures they had learned, but they had difficulties with their verbal explanations. In addition, none of the LAF students gave correct answers in the conceptual tasks.

At the pre-test time for the Pr2 problem (comparison of 1/2 and 4/5), of the 17 TAF students who responded correctly, 15 transformed fractions into the common denominator using the least common multiple to compare, whereas the other two students answered correctly without giving any justification. In the post-test task for the Pr2 problem, most students had developed verbalising thinking by applying conceptual strategies and a greater variety of strategies. Such strategies were used by 8 LAF and 21 TAF students, who answered the problem correctly. Specifically, two LAF students compared the fractions using equivalent fractions, and the remaining six students compared fractions using ½ and 1 as benchmarks, making frequent reference to the number line, which was recalled in their representations. Seventeen TAF students compared fractions based on their value on a number line [in addition, three of those drew fraction strips (see Figure 1) and fraction circles], three used the relationship between equivalent fractions, and one answered correctly by drawing fraction strips to show the difference between the sizes.

FIGURE 1



Using fraction strips to compare the fractions ½ and 4/5 (Pr2 task). Unsuccessful representation but correct answer

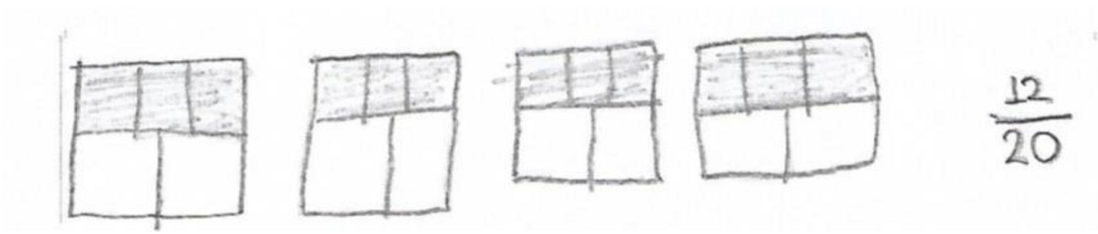
At the pre-test for the C3 task (fraction comparison between 1/7 and 1/8), only three LAF students answered correctly but without giving any justification. Of the 17 TAF students who answered correctly, 13 did not give any justification, while the other 4 gave explanations such as, 'Between two fractions with a common numerator and different denominator, the larger the denominator, the smaller the fraction'.

At the post-test, 9 LAF and 21 TAF students responded correctly to task C3. Eight out of 9 LAF students and 15 TAF students justified their answer using the value of unitary fractions on a number line (1/7 is closer to 1 than 1/8 on a number line), and one student justified the response by explaining, 'As we divide a pie into more equal parts, these parts are smaller in size (fractional units), so 1/7 is larger than 1/8'. In addition, five TAF students stated that, 'As we divide the unit into more equal parts, so the value of these parts is reduced', and one TAF student explained that 'In fractions with a common numerator and different denominator, the larger the denominator, the smaller the fraction'.

Pictorial representation of fraction multiplication. In problem Pr1, students were asked to draw a picture to represent a fraction multiplication operation, 4x3/5. At the pre-test time, only one LAF student attempted, unsuccessfully, to represent the problem's data by drawing (see

Figure 2; the multiplication as a repetitive addition functioned as a learning obstacle in this case). This representational ability was also low among TAF students; 33.3% of the TAF students (8 of 24) tried to provide a representation, but only 16.6% (4 out of 24) gave a correct representation.

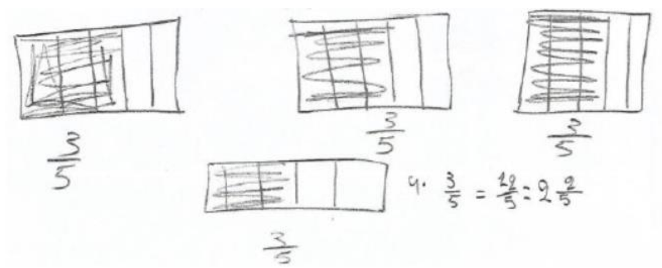
FIGURE 2



Unsuccessful response to Pr1 (4x3/5) by an LAF student

At the post-test time, 80% of LAF students and 91.7% of TAF students responded correctly. Of these, only 30% of LAF students and 75% of TAF students were able to draw correct images to represent the problem (Figure 3).

FIGURE 3



Successful response to Pr1 (4x3/5)

DISCUSSION

The results suggest large effect sizes in the students' overall performance for both groups in fractions knowledge and skills, one month after the explicit CRA intervention period. The intervention was equally beneficial for both LAF and TAF students, and the magnitude of gains is consistent with other studies' findings that implemented explicit instruction in fractions (Flores & Kaylor, 2007; Scarlato & Burr, 2002) or a CRA approach (Butler et al., 2003; Jordan et al., 1999). In addition, despite the initial significant differences between the LAF and TAF students, the gap was bridged after the CRA intervention.

Parsing out the specific effects of the CRA, the bigger influence of instructional intervention was observed in the domains of fraction placement (2/3) on a number line and problem-solving situations. Although the task of fraction placement on a number line was initially one of the most difficult for both groups, after the intervention, both groups of students improved significantly. This could be attributed to the fact that the number line was a key teaching tool in the study. The importance of representing fraction magnitudes by using a number line has been highlighted by researchers (e.g. Lemonidis, 2017; Siegler, Thompson, & Schneider, 2011).

Problem-solving tasks were initially difficult, especially for the LAF students. After the CRA intervention, both groups, but especially the LAF students, showed a considerable improvement. Further qualitative analysis showed that this impressive improvement for the LAF students could be attributed to their increased conceptual knowledge of fractions and better use of representation strategies that might lead to more effective linking of the fractions' operations to problem-solving situations. It is characteristic that none of the LAF students achieved that at the pre-test time, whereas 3 out of 10 used drawings to represent their solutions at the post-test. Niemi (1996) found that the level of representational knowledge predicted performance in problem-solving, justification, and explanation tasks in fractions.

In addition, the LAF students exhibited significant improvement in fraction procedural tasks when asked to execute the subtraction 9 - 6¼ and to convert 5¼ into an improper fraction. While, at the pre-test time, most students used exclusively algorithmic rules, after the intervention they applied a greater variety of strategies, both conceptual and procedural. This was an important change in the behaviour of both groups and a new skill among LAF students.

In brief, the most important effects of the explicit CRA instruction, especially among LAF students, was the conceptual understanding of fractions (e.g., by placing them on a number line) with a large effect size of .64, and the use of both conceptual and procedural strategies in problem solving. Moreover, the progress in pictorial representation of fractions among LAF students showed that they acquired flexibility in use of strategies. Flexibility and variety of strategies in fractions operations are key factors in the quality of mental calculations and number sense of fractions (Lemonidis et al. 2018; Newton, 2008; Yang, Reys, & Reys, 2009).

In general, the explicit CRA instruction yielded similar effect sizes for overall fraction performance between the TAF and LAF groups. This may not be accidental. The intervention was designed to cover primarily the learning needs of LAF students who fall behind in fractions. Most of the instructional coverage corresponded to the fourth-grade curriculum, and the intervention was intensive, lasting for 21 instructional sessions of 45 minutes, applied almost every day within a one-month period. In addition, as part of the explicit nature of the intervention, there was plenty use of a number line (concrete phase) and drawings modelled by the teacher, practiced by the students (representational phase). This approach was not without a price. Towards the end of the instructional period, some high achievers in the TAF group expressed sporadic complaints to their teachers about the prolonged instructional focus on fractions. The complaints were also recorded by one of the researchers who observed several instructional sessions. This raises an instruction efficiency issue about the focus and use of instructional time. It seems that the specific CRA intervention can be equally effective for both TAF and LAF students (Table 2) but the issue of who benefits most from choices regarding instructional focus on content and target population is a valid one. This critical issue might have not been adequately addressed in the relevant Universal Design for Learning (UDL) literature (Meyer & Rose, 2000; Rose & Meyer, 2002, 2009; Rose & Strangman, 2007). In our intervention programme, the target instructional group was low achievers in fractions and not typically developing students. In Greek classrooms, by contrast, the focus is the reverse; in typical instructional situations, mathematics instruction is mainly designed for typically developing students.

Limitations

The limitations of the present study include (a) the small sample size, which may restrict the power of the statistical tests to detect larger post-intervention changes and larger effect sizes; (b) some possible 'roof effects', especially for the TAF students at the post-test; and (c) the lack of testing for the maintenance and retention of acquired fraction skills among LAF and TAF students with a further delayed post-test after a period of some months. However, it is noteworthy that the post-test was administered one month after the end of the intervention, and this may suggest some evidence for the maintenance of knowledge and skills in fractions.

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