A comparative study of Geometry in first-grade Mathematics textbooks from four countries

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ABSTRACT
This study presents a comparative analysis of the treatment of geometry in first-grade mathematics textbooks from Greece, Cyprus, Austria and Singapore, countries with different levels of achievement in the PISA mathematics assessment. Data were collected through qualitative content analysis, focusing on the extent, sequencing and instructional aims of geometric content, as well as the nature of the proposed activities. The findings reveal differences in curricular priorities, with Greece and Cyprus placing greater emphasis on geometric content coverage, while Austria and Singapore prioritize problem-solving processes and the development of critical thinking skills.

KEYWORDS
Geometry, first-grade, school Mathematics textbook

RÉSUMÉ
Cette étude propose une analyse comparative de la place et du traitement de la géométrie dans les manuels de mathématiques de première année du primaire en Grèce, à Chypre, en Autriche et à Singapour, pays présentant des niveaux de performance différents dans l'évaluation PISA en mathématiques. Les données ont été recueillies au moyen d'une analyse qualitative de contenu portant sur l'étendue, la progression et les objectifs didactiques des contenus géométriques, ainsi que sur la nature des activités proposées. Les résultats mettent en évidence des différences significatives dans les priorités curriculaires entre les pays étudiés.

MOTS-CLÉS
Géométrie, première année, manuel scolaire de Mathématiques

INTRODUCTION

Many studies highlight the difficulties that students face internationally in mathematics. Especially in Greece, the negative course of mathematics school education for the Greek

education system is evident from the results and performance of Greek students in international evaluation programs of educational systems of countries, such as the PISA (Program for International Student Assessment), the OECD (Organization for Economic Co-operation and Development). In this international evaluation program, Greece is consistently below the average of the participating OECD countries, with the last two competitions of 2012 and 2015 ranking Greece in 42nd and 43rd place, respectively, in mathematics in a total of 72 countries (OECD, 2013, 2014; PISA, 2016).

The results of PISA and other international organizations are considered quite reliable, as a result of which they are the trigger for the effort to reform the education system of many countries. For Greece, 2021 is a milestone in education, as it is in the process of reforming its education system through the writing of new Curricula and textbooks. The previous reform took place in 2003 with the writing of the Curricula and the circulation of the respective textbooks in 2006.

Because it is not only Greece that faces the above problems, in the international literature there is a plethora of research that has dealt with the difficulties faced by students in mathematics. In particular, the international literature attributes students' difficulties in mathematics, including textbooks (Deliyianni et al., 2016; Dreher & Kuntze, 2015; Jiang & Chua, 2010; Shahbari & Peled, 2015), which are basic teaching aids used to acquire school knowledge having, however, in addition to their teaching and methodological mission and an essential role in shaping incentives for effective participation of students in the learning process. The present study, based on the above data, presents the findings from the comparative study of textbooks between countries that have achieved different levels in the PISA evaluation (2016) and in particular, focuses on geometric concepts.

THEORETICAL FRAMEWORK

Geometry in the First-Grade Mathematics

Geometry is considered a key object of study in mathematics, as it can help students to perceive the world in a systematic way (NCTM, 2000). It is even considered a field inextricably linked to mathematical knowledge (Atiyah, 2001). In particular, new approaches to teaching and learning geometry, such as the development of mathematical creativity, mathematical and critical thinking, are reflected in international research (Gridos et al., 2019; Gridos, Avgerinos, Deliyianni et al, 2021; Gridos, Avgerinos, Mamona-Downs et al., 2021; Kell et al, 2013; Levav-Waynberg & Leikin, 2012; Singer et al., 2017) that they are the modern way of reforming the teaching and learning of various mathematical concepts. At the classroom level, geometry, more than other areas of mathematics, is suitable for the development of mathematical and critical thinking, as it provides opportunities for exploration, allowing the smooth integration of multiple approaches into a problem (Herbst, 2002).

The choice of the field of geometry of the first-grade for the present comparative study of textbooks between countries that have achieved different levels in the PISA assessment was made given that students at this age come into contact for the first time with standard mathematics teaching and in particular for the concepts of geometry. Therefore, it is important to study the geometric concepts in this class from multiple perspectives (such as naming, description, recognition, distinction, grouping, and classification).

Textbook analysis has long constituted an important research strand in mathematics education, as textbooks serve as a primary link between intended curricula and classroom practice. Beyond conveying mathematical content, textbooks reflect pedagogical orientations, epistemological assumptions and instructional priorities, shaping both teaching approaches and students' learning opportunities. Large-scale comparative studies have demonstrated that

textbooks play a crucial role in structuring mathematical knowledge and guiding classroom activities across different educational systems (Schmidt et al., 2001; Valverde et al., 2002).

Within this tradition, comparative textbook research has been widely used to examine similarities and differences in content organization, task types and cognitive demands across countries. Such studies have highlighted how variations in textbook design are related to broader curricular goals and instructional emphases in mathematics education (Charalambous et al., 2010; Fan et al., 2013). In the field of geometry, international comparative analyses have further shown that textbooks differ substantially in the extent of content coverage and in the balance between conceptual understanding, procedural practice and problem-solving activities (Wang & Yang, 2016). Consequently, textbook analysis provides a valuable methodological framework for investigating curricular priorities and instructional approaches in different national contexts.

The Study

Aim of the study

The purpose of the present study was to record the similarities and differences in the first-grade mathematics textbooks from four representative countries: Greece, Cyprus, Austria and Singapore.

Research questions

In the present study the mathematics textbooks of the four countries are studied in relation to the following research questions:

(1) What is the extent of the teaching of geometric concepts in the mathematics textbooks of four countries: Greece, Cyprus, Austria and Singapore?
(2) In what time period are geometric concepts taught to primary school students in four countries?
(3) What is the goal setting for the geometric meanings in the mathematics textbooks in each of four countries?
(4) How much emphasis is placed on the naming of two-dimensional shapes? In the description in terms of edges and sides? In their classification based on their position, size and shape?
(5) Are there any open or closed questions in the relevant activities related to the geometric concepts in the first-grade mathematics textbooks in four studied countries?

METHOD

The research method followed in the present studied is the qualitative method with content analysis as a method of research strategy. “Content analysis is an approach to the analysis of evidence and texts that tries to quantify the respective content in a systematic and repeatable way based on pre-formed categories” (Bryman, 2017, p. 325). For the application of the method, first a qualitative recording is made and then some of the findings are coded and quantified, in order to facilitate and make sense of the conclusions. In order to ensure the objectivity of the method, special attention has been paid to the definition of such categories that enable anyone to repeat the process and come to the same conclusions.

Research sample

The textbooks selected in the present study will not be analyzed comprehensively, but their analysis will be limited to the sections and chapters that contain geometric concepts according to Table 1. At this point it is worth mentioning that the units of analysis were the images, the

activities, the applications, the inserts, the appendices, the exercises in the textbooks (student books and workbooks) of all four countries.

TABLE 1

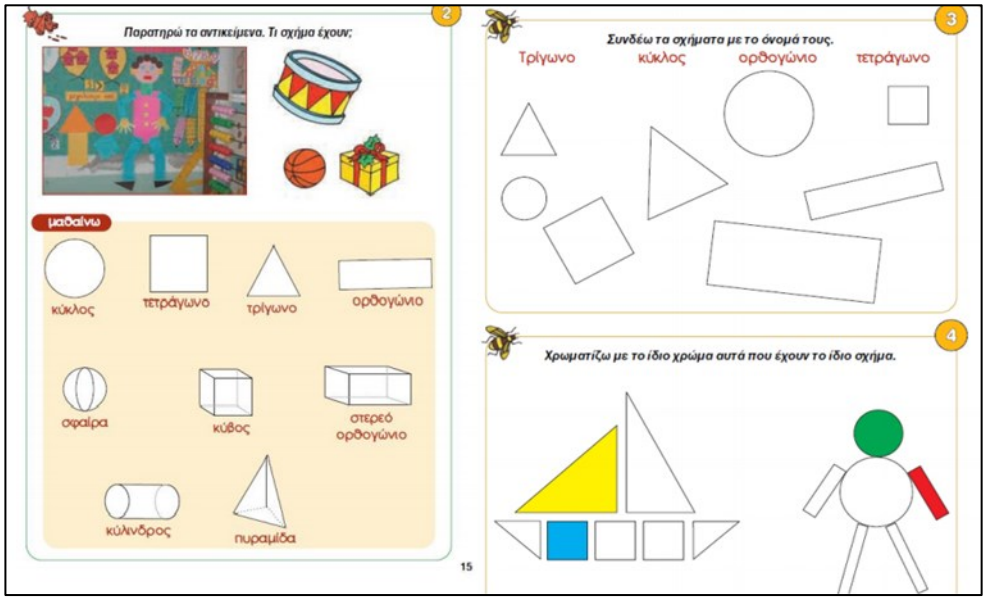
The research sample from the first-grade mathematics textbooks of four countries

	Chapters	**Modules**
Greece	15	6
Cyprus	14	3
Austria	4	2
Singapore	2	2

Geometry in Greece: First-grade Mathematics textbooks

In Greece in the 1st module of the 1st period (the school year is divided into three periods) the students practice first in the orientation in space (1st chapter) and immediately after (2nd chapter) in the recognition of the form of flat shapes and solids and their naming (Figure 1).

FIGURE 1



Plane and solid geometry in the first-grade Mathematics textbooks in Greece

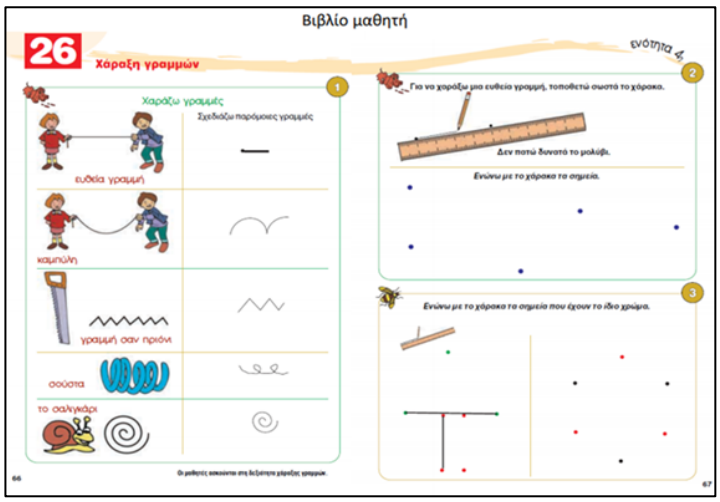
In the 4th module of the 2nd period the students practice drawing straight, curved and zigzag lines. In addition, they learn to use the ruler to draw a straight-line joining point (Figure 2).

In section 5, students are introduced to the Cartesian plane (squared paper) and learn to move on it taking into account both dimensions, horizontal and vertical.

In the 6th module, students practice the skill of recognizing the form and correct naming of the most basic shapes (flat shapes and solids) that surround them (Figure 3).

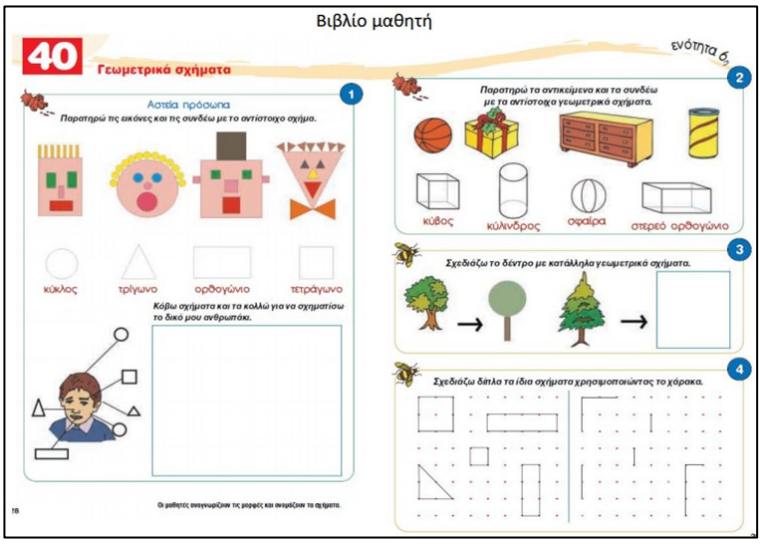
In the 7th module of the third and last period, students practice the acquisition of a skill related to the use of shape in geometric situations, a skill related to reading and using geometric shapes and complex images that result from the composition of geometric shapes. In addition, students learn to reconstruct a shape from its components, while cultivating the skills of visual analysis and completion of a mosaic (Figure 4).

FIGURE 2



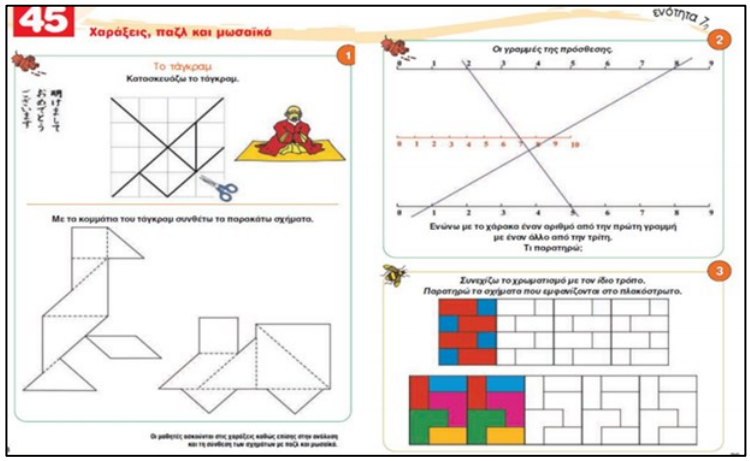
The drawing of lines in Greek Mathematics textbooks

FIGURE 3



The skill of recognizing the form of geometric shapes in the Greek student's book

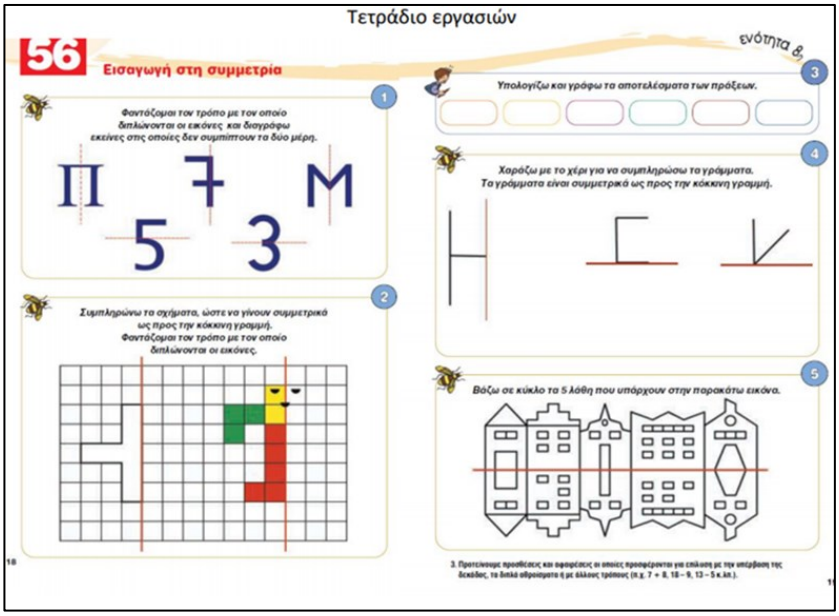
FIGURE 4



The drawing lines in the Greek student's book

In the 8th module of Mathematics, the students are introduced to the concept of symmetry on the axis. Students are introduced to the concept of the axis of symmetry, without, however, naming this phrase because of their young age. Instead, the phrase "the line separating the shape in half" is usually used (Figure 5).

FIGURE 5



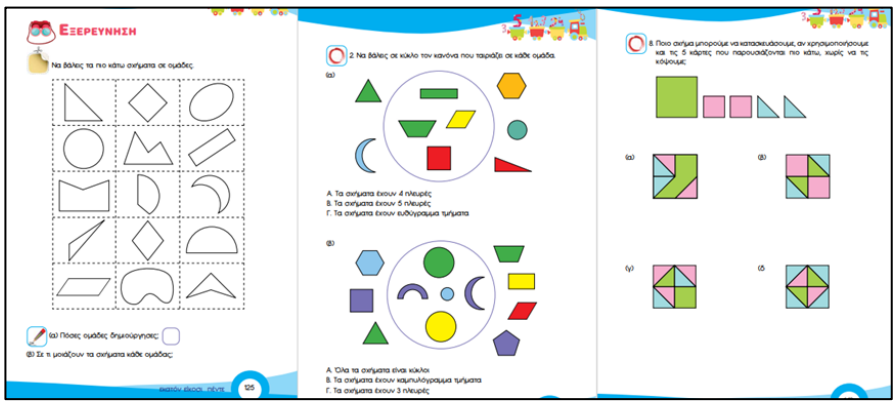
Introduction to symmetry in the first-grade student's workbook in Greece

The teaching of geometric concepts is completed in the first-grade with the 9th module, where students are further trained in drawing shapes, puzzle composition and paving.

Geometry in Cyprus: First-grade Mathematics textbooks

In Cyprus, geometric concepts are taught throughout the school year, i.e. in all three quarters, where in the first to the fourth module, which corresponds to five chapters, students identify, name and describe two-dimensional shapes (e.g. rectangle, square, triangle and circle), distinguish the basic characteristics of two-dimensional shapes (closed shape, number of sides, number of angles), compose and separate two-dimensional shapes into other sub-shapes (Figure 6).

FIGURE 6



The dimensional shapes in Cyprus first-grade Mathematics textbooks

In the seventh module, which corresponds to three chapters, students are introduced to solid shapes (cube, rectangle, sphere, cylinder, cone), learn to name them, describe them and relate them to objects in the environment (Figure 7).

FIGURE 7

The three-dimensional shapes in Cyprus first-grade Mathematics textbooks

In the eleventh module, which includes 7 chapters, students learn to develop the concept and procedures of measurement and measure the length of objects by repeating (conventional and non-conventional) units of length. For this reason, tools such as the ruler, the clip, the wool, etc. are used as depicted in the following Figure (Figure 8).

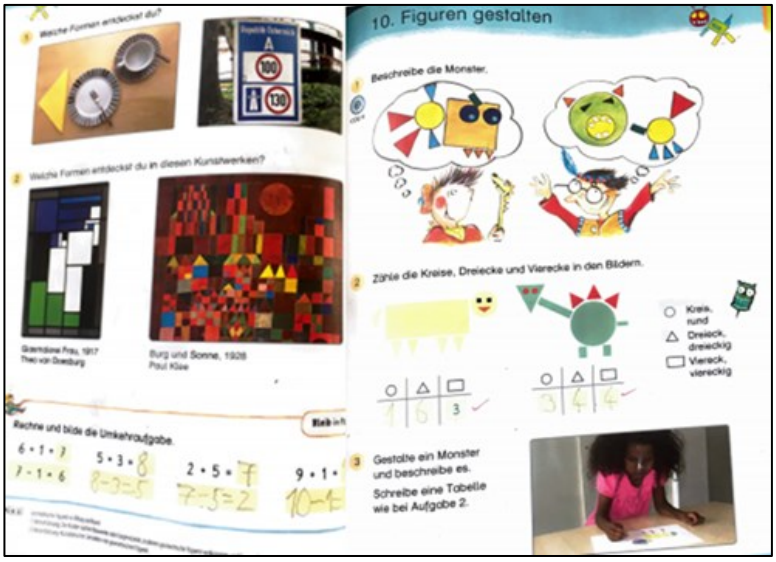
FIGURE 8

The measurement of length and perimeter in Cyprus first-grade Mathematics textbooks

Geometry in Austria: First-grade Mathematics Textbooks

The geometric concepts in Austria first appears in the fifth chapter where students practice measuring the length or height of various objects using their hands (palm and fingers) and feet. Before the end of the first teaching period (near Christmas) the students are asked in the 10th chapter to distinguish different shapes (circle, rectangle, triangle) from their daily life or from representations familiar to them (Figure 9).

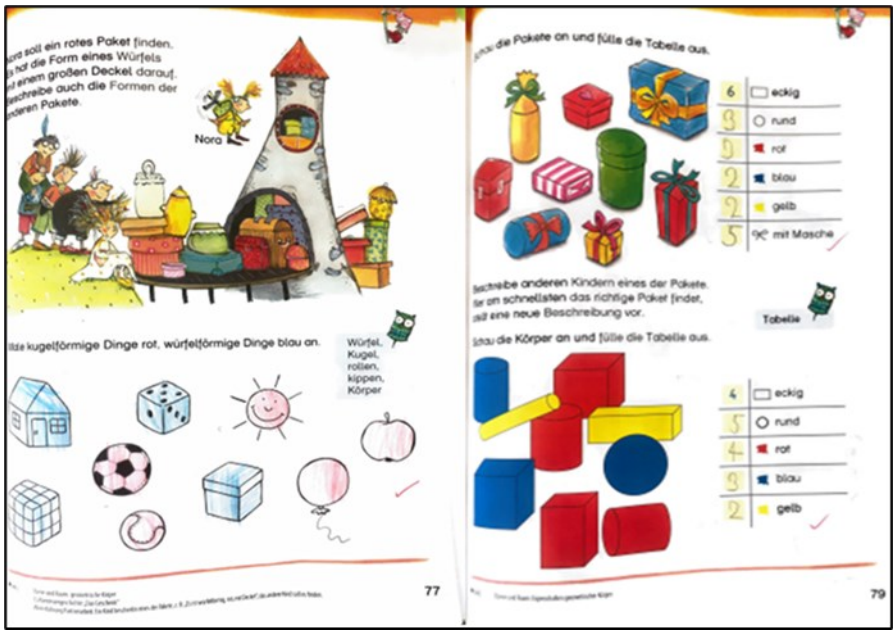
FIGURE 9



Recognition of shapes in Austria first-grade Mathematics textbooks

During the second teaching period and in the 19th chapter, the geometric concepts reappear, specifically the solids (sphere, cube, parallelepiped), where the students through various activities practice the recognition of the square and the circle on the (Figure 10).

FIGURE 10



Recognition of circle and square on solid shapes in Austria first-grade Mathematics textbooks

Geometry in Singapore: First-grade Mathematics Textbooks

The geometric concepts in the textbooks of Singapore are included in the first issue and in fact in the seventh (out of the nine) sections. In the eighth section, students learn for the first time the measurement of length and height through various activities. In the seventh unit in Singapore, students are introduced to the recognition of basic flat shapes (circle, triangle, square, rectangle) on shapes of everyday objects (Figure 11).

FIGURE 11

The recognition of flat shapes in Singapore first-grade Mathematics textbooks

Through shapes the students are involved in a variety of activities that include grouping, classification, patterns, drawing lines, filling in shapes, constructions. The measurement of length and height in the textbooks of Singapore is included in the 8th section of the first issue exclusively with non-conventional units of measurement such as paper clips, papers, feet, hands, etc.

RESULTS

The data collection from the content analysis of the first-grade textbooks in the four studied countries is depicted in the following table (Table 2).

TABLE 2

Summary of the findings

	Greece	**Cyprus**	**Austria**	**Singapore**
Percentage of Curriculum	24%	25%	11%	11%
Teaching Period	1st, 2nd, 3rd semester	1st, 2nd, 3rd semester	First (mainly) and second half schooling period	First half schooling period
Naming Shapes	√	√	√[1]	√
Description in terms of angles, sides and vertices		√	√	
Drawing lines	√	√		√
Recognition	√	√	√	√
Distinction	√	√	√	√
Classification	√	√		√
Grouping	√	√		√
Solid shapes	√	√	√	√
Open-ended activities		√	√	√

The following data were identified in more detail in the studied research questions:

(1) In Greece, geometric concepts extend to 15 chapters out of a total of 63 chapters (24%). In Cyprus geometric concepts extend to 3 of the 12 modules in the total didactic content (25%). In Austria they extend to 4 chapters in a total of 28 chapters (10.7%). In Singapore in a total of 18 modules, 2 of them refer to geometric concepts (11%).

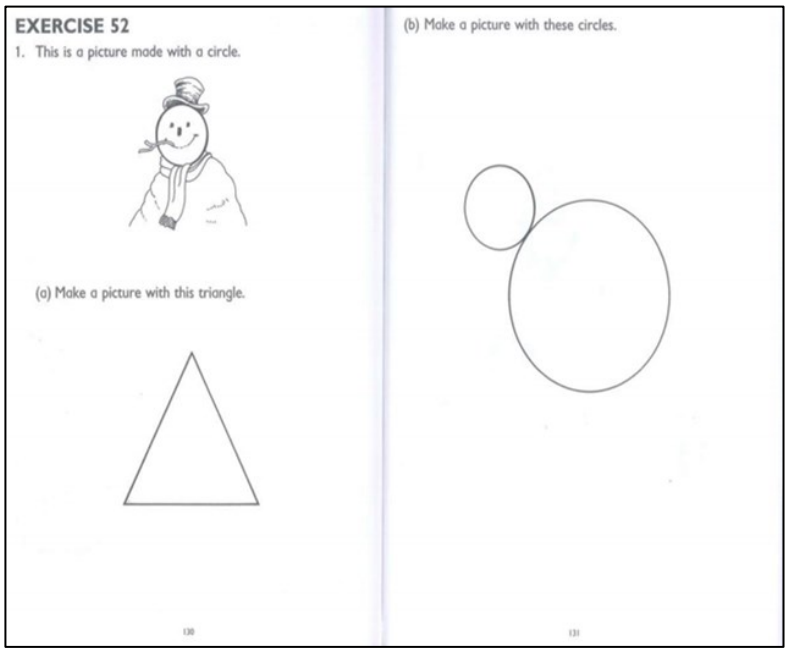
(2) In Greece, geometric concepts of the 15 chapters are infused in all three teaching periods of each school year. The same is recorded in Cyprus. In Austria geometric concepts of the 3 chapters are mainly contained in the first teaching period (where the school year consists of two teaching periods lasting about five months), while the nineteenth chapter, which deals with the recognition of the rectangle and the circle, is intended to be taught in the second teaching period. In Singapore, geometric concepts take place at the end of the first teaching period (7th and 8th module), as algebraic concepts are prioritized throughout the previous period.

(3) The goal setting in Greece is extensive starting from the orientation in the field, the recognition of the form and the naming of the flat and solid shapes during the first teaching period. In the second course, it aims to acquire the skill of drawing straight, curved and zigzag lines, orientation on squared paper, drawing flat shapes on squared paper, grouping shapes based on their shape. In the third course students are trained in the ability to use shapes in geometric situations, in the ability to visually analyze and complete a mosaic and use a ruler to draw lines of basic levels of shapes. Finally, students make a first experiential contact with objects, images and shapes that are symmetrical regarding their axis. In Cyprus, the goal of geometric concepts includes the description and construction of various types of lines, the identification, naming, description and classification of two-dimensional and three-dimensional shapes, the composition and separation of two-dimensional shapes into other components, measuring length and perimeter with conventional units of measurement. In Austria the goal setting of geometric concepts is limited to the distinction and recognition of basic levels of shapes in various representations with a simple reference to solids, mainly at the naming level. In Singapore, students gain a fairly rich first experience with basic level shapes where, in order to identify and distinguish themselves, students engage in a variety of mathematical activities, most of which aim at meaningful mathematical communication with each other and contact with surrounding objects.

(4) The naming of shapes has a different emphasis in the books of the first-grade textbooks from country to country. Specifically, in Greece students learn to name both flat shapes (circle, square, triangle and rectangle) and solid bodies (triangular pyramid, cube, rectangular parallelepiped / solid rectangle, cylinder and sphere). The same is true in Cyprus as in Greece, as here students learn the names of flat shapes and solids. In Austria students learn the use of the terms circle and rectangle through the activities presented above. As for the solid shapes, their names are mentioned (sphere, cylinder, cube, triangular pyramid, rectangular parallelepiped) but without special emphasis on their memorization. In Singapore, flat shapes are named from the beginning of the unit, when students are asked to identify them in familiar representations. The flat shapes whose name the students learn and become familiar with are the circle, the triangle, the square and the rectangle. As for the description in terms of angles, vertices and sides, this is observed in the books of Austria in a description activity and in the books of Cyprus where in activities of distinguishing shapes (levels) students are asked to recognize if they are closed shapes, the number of their sides and corners. All countries, except Austria, include activities related to the classification of shapes by size, position and shape.

(5) Regarding the type of questions that are asked in the activities of the first-grade mathematics textbooks here is a recorded difference. Analytically, in Greece all questions are close-ended and need one singular answer. On the contrary, in the textbooks of Cyprus, precisely because solving complex problems is a priority, most activities are open-ended exploration with rich material to be exploited. In Austrian textbooks there is a combination of open-ended and closed-ended questions on student activities. The open-ended questions refer to the chapter of measuring length and height, while for the shapes the questions are all closed-ended. In the textbooks of Singapore, the activities in which the students are involved are mainly open-ended, since, after all, the discussion based on geometric questions and the development of thinking through a view of possibilities is a priority. An example is set below (Figure 12).

FIGURE 12



Example of an open-source activity in Singapore Primary School Mathematics textbooks

RESTRICTIONS OF THE STUDY

In the present study, first-grade mathematics textbooks from four countries were selected based on their relevance to the research aims and the authors' access to the material. Nevertheless, the findings derived from the content analysis of geometric concepts should be interpreted with caution, as they cannot be readily generalized beyond the specific textbooks examined. It remains an open question whether the observed characteristics are representative of all textbooks within each national context. A further limitation concerns the focus on early primary education, which does not allow for direct inferences about students' mathematical development at later educational stages

CONCLUSIONS

The present study examined the treatment of geometric concepts in first-grade mathematics textbooks from four countries. The comparative analysis revealed notable differences with

respect to the extent, sequencing and instructional emphasis of geometry across the examined textbooks. In particular, geometry occupies a larger proportion of the curriculum in the textbooks of Greece and Cyprus, accounting for approximately one quarter of the total content, whereas in the textbooks of Austria and Singapore geometric concepts are more limited in scope, representing about one tenth of the curriculum. Furthermore, in Austria and Singapore the introduction of geometry is relatively delayed, as priority is given to other mathematical domains during the initial stages of instruction. As related to the open-ended activities, it is found that the textbooks of Cyprus and Singapore are flooded with such activities, while in Greece they contain exclusively close-ended activities. Moreover, Singapore seems to perform well on international assessments due to the combination of visual to other representations forms (Wang & Yang, 2016). In Austria there is a combination of open and close-ended activities.

From the analysis of the priority of the goals that are set, it is found that in all the textbooks the goal is the recognition and distinction of flat shapes and solid bodies. The description in terms of corners, sides and edges is found only in the books of Cyprus and Austria, while there are no such references in the textbooks of Greece and Singapore. In terms of grouping and classification activities of these shapes they are included in the books of Greece, Cyprus and Singapore. Of particular interest is the naming of shapes where in Greece students learn to name both flat shapes (circle, square, triangle and rectangle) and solids (triangular pyramid, cube, rectangular parallelepiped / solid rectangle). Likewise in Cyprus, where students learn the names of flat shapes and solids. The activities presented above are used in Austria to teach students the use of the terms circle and rectangle. Regarding solids while their names are mentioned (sphere, cylinder, cube, triangular pyramid, rectangular parallelepiped) it is with no special emphasis on their memorization. In Singapore, flat shapes are named from the beginning of the unit, when students are asked to identify them in familiar shapes. The flat shapes whose name the students learn and become familiar with are the circle, the triangle, the square and the rectangle.

Overall, the findings suggest different curricular orientations in the teaching of geometry at the first-grade level. While some textbooks emphasize broader exposure and explicit treatment of geometric concepts, others place greater weight on the development of problem-solving processes through exploratory and representationally rich activities. In this context, the integration of meaningful, context-based geometric tasks may further support students' conceptual understanding, as suggested in previous research (Julie, 2013; Van de Walle, 2007; Wang & Yang, 2016). These observations may inform future textbook design and curriculum development in early mathematics education.

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