A review of guidance and structure in Elementary School Mathematics instruction

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ABSTRACT

Guidance and structure have both been linked to higher achievement, but the two terms are not clearly defined and, thus, are used interchangeably. This makes it difficult to determine the practical implications of interventions and how teachers should apply guidance and structure in their own classrooms. This paper defines and differentiates guidance and structure in elementary school mathematics research. Specifically, guidance involves interactive and responsive student-teacher interactions during teaching while structure refers to the explicitness of the lesson plan, curriculum, or materials. We make this distinction because guidance provided by teachers requires a higher level of expertise and preparation from the teacher. We found some support for the benefits of guidance, with the caveat that teachers should consider individual student factors, such as prior knowledge. Structure encompasses a wider variety of activities, including worked examples, ordering problems from easy to difficult, or providing formula sheets during problem solving.

KEYWORDS

Instructional guidance, elementary school, mathematics, guidance, structure

RÉSUMÉ

L'orientation pédagogique et la structure ont toutes deux été liées à des résultats supérieurs, mais les deux termes ne sont pas clairement définis et sont donc utilisés de manière interchangeable. Cela rend difficile la détermination des implications pratiques des interventions et la manière dont les enseignants devraient appliquer les orientations pédagogique et la structure dans leurs propres classes. Cet article définit et différencie l'orientation pédagogique et la structure de la recherche en mathématiques à l'école primaire. Plus précisément, l'orientation pédagogique *implique des interactions entre élèves et enseignants interactives et réactives pendant l'enseignement tandis que la structure se réfère à l'explicite du plan de cours, du programme ou du matériel. Nous faisons cette distinction parce que les orientations pédagogiques fournis par les enseignants exigent un niveau plus élevé d'expertise et de préparation de la part de l'enseignant. Nous avons trouvé un certain soutien pour les avantages d'orientation pédagogique, avec la mise en garde que les enseignants devraient tenir compte des facteurs individuels de l'étudiant, tels que les connaissances antérieures. La structure englobe une plus grande variété d'activités, y compris des exemples pratiqués, la commande de problèmes de facile à difficile, ou la fourniture de feuilles de formules pendant la résolution de problèmes.*

MOTS-CLÉS
Orientation pédagogique, école primaire, mathématiques, orientation, structure

Introduction

Teachers have a tremendous amount of work to do and little time to do it. Fortunately, one of the most immediately applicable products of educational research is the ability to provide information to teachers about where to best expend their valuable energy and time. This article is specifically interested in the research regarding when teachers should implement guidance and structure during their lessons.

Teachers can spend significant amounts of time and energy providing guidance to students with the goal of improving learning. The reason guidance requires a high expenditure of time and energy is because highly guided instruction requires a substantial amount of preparation and a higher level of expertise than less guided activities, such as independent seat work or computer instruction (Gerard, Matuk, McElhaney & Linn, 2015). Similarly, teachers can spend substantial amounts of valuable time structuring their lessons, for example, by ordering problems on a worksheet from easier to more difficult, or by including prompts to have students write an explanation for their answers. For highly structured lessons, the effort is employed upfront before the lesson takes place. During the lesson the teacher can allow students to work alone (low guidance) or can be present and circulate the classroom providing instruction to students (high guidance). Although teachers are often encouraged to provide plenty of guidance and structure, the problem is we do not really know when guidance and structure are best employed during instruction. If teachers have information about the best time for high guidance and high structure, their lesson planning and instruction can be more efficient.

The reason we do not know when guidance and structure are best employed during instruction is because guidance and structure are not consistently defined and, as a result, research has not provided a clear picture of their effectiveness. Research that uses the term guidance has included guidance as both student-teacher interaction (e.g., Mayer, 2004; Terwel, van Oers, van Dijk & van den Eeden, 2009) and the structured use of materials and content (e.g., Baroody, Purpura, Eiland & Reid, 2015; Chen, Kalyuga & Sweller, 2015), which are quite different. We must separate the findings of research on student-teacher interactions from the findings of research on the structured use of materials and content.

Because of the absence of clear, consistent definitions of guidance and structure, it is difficult to determine their efficacy. The purpose of this paper is to point out the problems that have arisen due to the absence of consistent definitions of guidance and structure and to provide clear definitions that can be used for future research. We will first describe the diversity of studies that purport to research guidance and the implications of a lack of a clear definition of guidance. Then, we propose a clear definition of guidance that distinguishes it from structure, as well as a clear definition of structure that distinguishes it from guidance. Finally, we review research in elementary school mathematics on guidance and structure using these clear and precise definitions to model this new approach to defining and differentiating between guidance and structure. In our review, we include studies on guidance that not only reported studying guidance but also correctly met our definition of guidance. Therefore, studies that reported studying guidance but did not meet our definition of guidance (i.e., met our definition of structure) are included in the review of structure. We chose to limit this review to elementary school mathematics because the effectiveness of guidance may differ as a function of domain and because it is likely that as students learn and mature the effectiveness of guidance will change. Limiting the scope of the review avoided potential age and domain confounds.

A variety of definitions of guidance

The lack of a clear definition of guidance was illustrated in a 2006 article by Kirschner, Sweller, and Clark as well as the subsequent commentary articles. In their review, the authors argued that instruction that involved low guidance is less effective than instruction that involved high guidance for improving mathematics learning. Kirschner et al. (2006) classified low guidance instruction broadly, including studies that implemented constructivist, discovery, problem-based, experiential, and inquiry-based learning. These studies were classified as low guidance because they required the learner to construct and discover information independent of the teacher. It was concluded that low guidance instruction does not provide novice learners with sufficient information to set up and solve problems alone. This article provoked multiple responses that pointed out the considerable variability in the amount of guidance in the studies categorized as low guidance and, subsequently, problems with the conclusions.

There are other instances where guidance has been described as including a broad range of characteristics. Alfieri, Brooks, Aldrich and Tenenbaum (2011) compared unassisted discovery learning, which involved no feedback during learning, with explicit instruction, in which students were given feedback during instruction, which the authors described as high guidance. In a second meta-analysis, Alfieri et al. (2011) compared "enhanced discovery learning with guidance," defined as the use of self-explanation, to what they described as "other types of discovery learning". As with the work of Kirschner et al. (2006), both of these meta-analyses defined guidance broadly and collapsed multiple constructs into a single construct of guidance (e.g., combining teacher scaffolded instruction with teacher elicited student explanation). The absence of clearly operationalized variables makes it difficult to draw conclusions about the effectiveness of guidance and, furthermore, to provide information to teachers about best practices for utilizing high guidance versus low guidance. As another example, Baroody et al. (2015) categorized guided discovery learning into four categories: highly guided, moderately guided, minimally guided, and unguided instruction. Guidance in this study was broadly defined, involving several constructs that could be categorized as structure, including scaffolding and organization. Thus, as used in the current literature, guidance is poorly defined, not consistently defined, and frequently confounded with structure.

Guidance and structure defined

Guidance Defined

For this paper we base our definition of guidance in the social constructivist theory of Vygotsky (1962, 1978), which focuses on the co-construction of knowledge by teacher and student. Guidance is defined as the interaction between teacher and students, specifically, the amount of feedback teachers provide in response to students' questions and learning difficulties, the quantity and quality of teachers' responsiveness to students' questions and concerns, scaffolding provided by the teacher (i.e., not provided by worksheets or materials), and how often teachers ask students questions that are designed to cause students to think more deeply. For the purposes of this article, guidance refers only to student-teacher interaction and not peer interaction or parent-student interaction, as the nature of these interactions are different from teacher-student interaction and beyond the scope of the current review. In addition, while nonverbal cues and gestures from the teacher could be considered aspects of responsive teacher guidance, the research that has investigated cues and gestures does not exemplify the interactive nature of guidance as we have defined it.

Our definition of guidance aligns with Mayer's (2004) definition of guided instruction, as he distinguished between "pure discovery" learning and "guided discovery" learning, where guided discovery learning includes teacher provided guidance focused on the learning objective through hints, direction, coaching, feedback, or modeling. For the purposes of our review, we created four levels of guidance including high guidance, moderate-high guidance, moderate-low guidance, and low guidance. Table 1 shows the categories of the levels of guidance as we describe them here and the criteria for inclusion in each category.

Table 1

Categories of Guidance with Examples

	Description	Example
High Guidance	Collaborative construction of knowledge, teacher and student involved; lots of dialogue; substantial interaction; supports deep learning	Combination of several: co-construction; monitoring and providing assistance; feedback and responding to questions; opportunities for reflection
Moderate-High Guidance	Some interaction but not as responsive to students' needs as high guidance; more feedback and/or prompts than just Yes/No, Correct/Incorrect; prompts for student to talk	Scripted co-construction; feedback + this is the correct answer or what do we do next
Moderate-Low Guidance	No more than simple feedback; no elaboration; nor prompts to student; minimal interaction	Accuracy feedback from the teacher or computer
Low Guidance	No interaction; no intervention/dialogue other than to direct students to task	Solo work on worksheets, non-interactive lecture

We categorized guidance as high when there was substantial student-teacher interaction where the teacher was responsive to students' learning needs during a lesson or during problem solving and in which teachers supported the deep thinking of concepts. Examples of high guidance instruction include a teacher monitoring student responses during problem solving and providing assistance as needed, teachers providing feedback and responding to questions from students, teachers asking students questions for students to respond to, and teachers creating opportunities for reflection based on students' performance and needs. High guidance is effortful on the teacher's part. It requires full attention as teachers monitor student progress and respond to student needs during instruction. We included two moderate levels of guidance to encompass studies that were neither high nor low guidance. Moderate-high guidance included instances of corrective feedback in which the teacher could provide more information on why an answer is correct or incorrect, provided information on how to find the correct answer, or provided a correct answer. This differed from high guidance as high guidance included more elaborate feedback and in-depth conversation, as well as student-teacher interaction focused on deep thinking of concepts. Moderate-low guidance included instances of scripted feedback that provided nothing more than accuracy information to the student. Low guidance was indicated when the teacher and students were not actively engaged in discussion and problem solving, such as non-interactive lecture or students working alone.

Structure Defined

Whereas the definition and categories of guidance focus on the quantity and quality of student-teacher interactions during instruction, structure is defined here as the purposeful explicitness and organization of the lesson plan, curriculum, or materials for understanding. This is consistent with Miller's (1980, p. 163) definition of structure as "the purposeful ordering or placement of people, materials, and resources in time". This definition of structure separates guidance, that is the quantity and quality of teacher-student interaction, from structure as defined as the presence or absence of an instructional component, such as worked examples or ordered problems.

For the purposes of this study we created two categories of structure: low structure and high structure (see Table 2). Proponents of highly structured instruction (e.g., Bruner, 1960; Cobb, 1995) highlight the need to support less mature working memory or limited prior knowledge through the use of explicit and well-organized materials. Highly structured lessons can include instruction in which problems are ordered from easy to difficult or worksheets that scaffold learning with steps or ordered tasks. Worked examples or formula sheets are also good examples of structure. Static, instructional components not influenced by students, such as instructions directing them to certain aspects of the lesson are also examples of structure, as these instructions do not provide the same guidance as questions to students (Sidney & Alibali, 2015). Proponents of low structure (e.g., Piaget, 1977) argue that students benefit from less structure because it forces them to construct their own knowledge. Low structure, in this paper, refers to instruction without plans as described above. Items are not presented in any specific order and students must organize any information that is presented. Students are not provided scaffolding, such as partially completed problems or ordered problems. Low structure could take the form of students working on worksheets that are not organized in any way by the teacher. It could also involve having students decide how to set up problems as opposed to being given the steps to set up the problem.

Table 2

Categories of Structure with Examples

	Description	Example
High Structure	Explicitness and purposeful organization of the lesson plan, curriculum, or the use of materials	Worked examples, formula sheets, ordering problems to elicit understanding of rules (e.g., add-1, doubles)
Low Structure	Lack of explicitness or organization of a lesson, curriculum or materials	Students create their own formulas, representations, or solutions with no material aids. Students left to decide how they will set up or approach a problem

Distinguishing between high and low structure can be difficult. Most lesson plans include some level of structure, otherwise they would not be lesson plans. Typically, studies on structure compare some aspect of structure with a control group. For example, they could include worked examples versus no worked examples, or ordered problems versus randomly displayed problems. Some studies, however, compare different forms of structure.

Literature review

Approach and Methodology

The literature we identified came from searches within two major education databases: Educational Resources Information Center (ERIC) and PsycInfo. Several criteria were followed. First, we included studies in elementary school (i.e., up to grade 5) mathematics education. We limited the age range and topic area because the level of teacher guidance and lesson structure can have different effects depending on the subject, topic, and age group, making it difficult to draw conclusions. One study was kept that included sixth graders as it also included fifth grade students. We only included studies with an experimental design that manipulated different levels or types of guidance or structure, therefore case studies of new program implementations were not included. Articles on parent guidance or peer guidance were excluded as these are different from teacher guidance, which is the focus of this paper. Finally, we only used studies that had some measure of mathematics learning; studies focused solely on motivation or other variables were not included.

Our search was limited to English-speaking, peer-reviewed journals published after 1996. We implemented separate searches for guidance and structure. In our first search for guidance the following key terms were used: "mathematics and guidance" with the limiters "Elementary Education" for ERIC and "Childhood" for PsycInfo. Of the 111 articles this search produced, 37 were irrelevant as they discussed topics such as test anxiety, parent guidance, motivation, achievement gaps, and vocational guidance; 24 investigated new curricula and programs; 14 investigated teacher education; 13 were not elementary school mathematics topics; eight were not experimental studies; seven investigated methodology and test development; and one was a duplicate. Therefore, only seven articles met our search criteria. Given the low number of applicable studies we expanded the search using the terms 'mathematics' and 'guided and instruction' with the same limiters. Of the 76 articles this search produced, 29 investigated new curricula and programs but did not include an experimental design; 16 investigated teacher education or professional development; 12 were irrelevant as they discussed topics such as achievement gaps, were reviews of the literature; six did not involve elementary school populations; three investigated methodology and test development;

one was a duplicate; and one was not in English. Therefore, eight studies from the second search met our criteria and were relevant to our search criteria. Combined, the two searches of the guidance literature resulted in 15 studies that met our criteria. Of those 15 studies, five were reclassified as structure based on our criteria and were reviewed in the structure section.

In our search for structure we used the same databases and limiters with the terms 'structure' and 'mathematics' and 'achievement'. This brought up 184 articles many of which were irrelevant because they focused on family or goal structure; to deal with this we revised the search to include NOT 'family structure' NOT 'goal structure'. Of the 167 articles this search produced, 61 were related to topics other than mathematics achievement including school/class structure, student-teacher relations, motivation; 43 were related to psychometric research including factor structure of mathematics skills and test development/validation; 14 did not investigate elementary school mathematics; 13 were not in English; 13 compared conditions that manipulated different content or problem features not related to lesson structure; six examined family variables such as parent involvement; five were review articles or reports; four explored teacher variables such as pay and education; two were comparisons between the United States and other countries; and two investigated a program or curriculum without a comparison group. Therefore, seven of the 167 articles produced by the search fit our search criteria. Our literature review on structure includes these seven studies plus the five studies reclassified as structure from guidance. In addition, one study by Terwel et al. (2009) manipulated both guidance and structure. We discuss the results of this article in both the guidance and structure sections.

To organize our literature review, we first review studies that compared different levels of guidance. Next we review studies that compared different types or levels of structure. In our review of structure articles, we denote the five articles found in our guidance search with an asterisk.

Guidance Studies

The ten studies we identified as guidance used terms such as cooperative learning, guided inquiry, guided play, questioning, feedback, prompts, and co-construction techniques in their description of the teacher-student interactions. In each of these studies, the authors compared at least two conditions that differed in the type of guidance. Below we discuss these studies, focusing on how the authors described their conditions and classifying the conditions as high, moderate-high, moderate-low, or low guidance based on our own categories of guidance.

Terwel et al. (2009) compared the impact of two problem-solving lessons on student learning of percentages and graphs. In the condition they labeled as high guidance, fifth grade students were taught through the process of guided co-construction; students and teachers created representations of the percentages through teacher-initiated, guided discussions. In the low guidance condition students were provided with ready-made, completed representations and were not engaged in discussion with the teacher. Controlling for pretests scores, children in the high guidance condition performed better on a posttest and transfer test. This provided support for guided, interactive teaching when students are learning problem solving strategies for percentages and graphs.

Sengupta-Irving and Enyedy (2015) also compared a high guidance condition to a low guidance condition. Specifically, the study compared instruction labeled as "guided" to instruction labeled as "unguided" with fifth grade students learning data analysis and statistics. In the guided condition the teacher led students through the problem solving process with the teacher defining the problem and then leading students through the problem via interactive discussion. In the unguided, open approach, students completed the problem without any assistance from the teacher. They found no group differences in learning outcomes between conditions on data analysis and probability.

Similar to the first two studies, Fisher, Hirsh-Pasek, Newcombe and Golinkoff (2013) described guided instruction as a collaborative construction by students and teachers. In their study, Fisher et al. (2013) taught preschool students properties of shapes in three conditions: free play in which student activity was self-directed with no goals for learning, a guided play condition described as discovery learning with the presence of an active teacher participant, and an instruction condition in which the student observed the instructor talking through the material. For this study the free play was categorized as low guidance instruction as the teacher played no role. The guided play and instruction conditions included the exact same script, however, they were different in that the guided play included prompts and questions directed at the student to elicit participation during the lesson (moderate-high guidance) while in the instruction condition the teacher read the script while the student watched rather than asking the student for input (low guidance). The authors found that students in the guided play (moderate-high guidance) showed improved understanding of shapes over the other two conditions (both low guidance), and those improvements were still observed one week later. They found that for understanding properties of shapes, moderately high guidance, even when scripted, was better than instruction that involved the student passively listening to the teacher or playing alone without any guidance.

Carbonneau and Marley (2015) also worked with preschool students to compare the impact of different levels of guidance. The study investigated the impact of guidance on students' conceptual and procedural knowledge on a quantity discrimination task (which side has more) using manipulatives. In their study, the researcher would make two piles of objects and the child would have a crocodile mouth with instructions that the crocodile should eat the bigger number. After making the piles the researcher would ask, "Which one should the crocodile eat?". In one condition, which the authors labeled high guidance, after the child pointed to the pile the crocodile should eat the researcher would then read the number sentence represented by the piles and crocodile and correct the child if necessary. In the other condition, which the authors labeled low guidance, the researcher prompted the student to read the number sentence. Based on these descriptions and our definition and categories of guidance, both conditions were examples of moderate-high guidance because the researcher interacted with students and prompted students to talk, but did not give feedback beyond just correct or incorrect. Furthermore, there was not the collaborative construction of knowledge with a lot of dialogue we see in high guidance instruction. Carbonneau and Marley (2015) found that students who heard the teacher repeat their explanations and were corrected on their errors improved their conceptual and procedural knowledge more than students only received prompts to recite the number sentence on their own.

Fyfe, Rittle-Johnson and DeCaro (2012) examined the impact of prior knowledge on feedback during problem solving. Low performing second and third grade students worked on problem solving on mathematics equivalency problems with one of three types of feedback: no feedback, outcome feedback (i.e., correct or incorrect answer), or strategy feedback (i.e., correct or incorrect strategy used). Accuracy feedback tells the student if the final answer is correct or incorrect, while strategy feedback tells the student if the process they used to get the answer is correct or incorrect. All conditions were followed with brief, conceptual instruction. Based on our definition, the no feedback condition was categorized as low guidance because there was no feedback or interaction between the teacher and student, while the outcome and strategy feedback conditions were both categorized as moderate-low guidance because they received non-elaborative feedback on performance. The authors found that performance interacted with prior knowledge. Students with low prior knowledge in the two feedback conditions (moderate-low guidance) performed significantly better on a procedural knowledge posttest compared to the no feedback (low guidance) condition. Students with moderate prior knowledge in the no feedback (low guidance) condition performed significantly better on a procedural knowledge posttest compared to the feedback (moderate-low guidance) conditions. These results suggest that students with no prior knowledge benefit from more guidance, but this is not the case for students with moderate prior knowledge.

Building on this work, Fyfe, DeCaro and Rittle-Johnson (2015) examined the impact of working memory on different forms of feedback on equivalency problems. In their study, they gave second and third grade students accuracy-only feedback (classified as moderate-low guidance) or strategy feedback (also classified as moderate-low guidance). Fyfe and her colleagues found students with lower working memory benefitted more from accuracy feedback than strategy feedback, while students with higher working memory benefitted from both types of feedback. These results show that, like prior knowledge, working memory affects students' ability to utilize different types of feedback. In this case, although both forms of feedback were categorized as moderate-low, the low working memory group did not benefit as much from the strategy feedback as they did from the accuracy feedback.

Fyfe and Rittle-Johnson (2016) investigated the impact of computer feedback on second grade students' learning of equivalency problems. There were three conditions within computer-based problem solving: no-feedback; immediate accuracy feedback after each problem; and summative, accuracy feedback after all 12 problems were solved. We categorized the no-feedback condition as low guidance and the two feedback conditions as moderate-low guidance. Within each of these three conditions students were grouped as having high or low prior knowledge. The impact of feedback differed as a function of prior knowledge. Students with lower prior knowledge, performed better in the feedback conditions than no feedback conditions on solving equivalency problems. For students with higher prior knowledge, all conditions resulted in improvement on solving equivalency problems.

Kroesbergen and Van Luit (2002) compared guided instruction to structured instruction of multiplication for low-performing students in both special education and regular education classes. The authors defined guided instruction as teacher supported learning through the use of teacher-generated questions and problems selected based on the students' performance during the lesson. Structured instruction, in contrast, involved teachers following a clear lesson plan in a prescribed order for each lesson, but without opportunities for the teacher to prompt learning through questions or adjusting the instruction based on performance. Based on these descriptions, we categorized the guided instruction condition as high guidance instruction and the structured instruction condition as low guidance instruction. The study also implemented a control condition, which involved no extra instruction to the students' regular mathematics curriculum. Students in both treatment conditions improved on their problem solving skills more than students in the control condition. The high guidance instruction condition resulted in more improvement for multiplication problem solving than the low guidance condition for students in both regular and special education classes, but especially for students in regular education classes. Students in special education classes improved more in the low guidance condition in comparison to the high guidance condition on automaticity with multiplication problems, which may be the result of the cognitive load and is consistent with the work of Fyfe et al. (2015).

In another study, Kroesbergen and Van Luit (2005) taught elementary school students (age not specified) with mild intellectual disabilities strategies for multiplication over four months. They compared a high guidance condition (labeled guided instruction) with a moderate-low guidance condition (labeled direct instruction). The high guidance condition involved discussions between the instructor and students on multiplication solution procedures. Specifically, the teacher focused on the learning difficulties students were having and initiated discussions with the students by asking and answering questions related to the particular difficulties. The moderate-low guidance instruction involved the teacher instructing students on multiplication solution strategies, but students did not have the same opportunities to ask and answer questions. Teachers could give feedback about the correctness of the strategy being used but no other feedback was given. The authors found that students in both conditions improved on measures of multiplication automaticity and ability, but students in the low guidance condition showed greater improvement, supporting earlier findings that direct instruction involving less guidance was advantageous when teaching students with mild intellectual disabilities (Kroesbergen & Van Luit, 2002).

Moreno and Durán (2004) compared whether adding verbal instructions to graphic representations of problem solving on a number line improved the performance of fifth and sixth graders' problem solving. All students in the study had low prior knowledge on solving addition and subtraction problems, but had varying computer experience. In the study, students were shown a -9 to +9 number line with a bunny that moved along the line. For each problem, the problem solution was shown by moving the bunny along the number line. One of the two conditions included the addition of a verbal explanation (moderate-low guidance) whereas the other condition included no verbal explanations (low guidance). Students with high computer experience who received the verbal explanations performed better than all other students on the posttest. There was no main effect between the two conditions.

Guidance summary. Of the ten studies discussed seven studies (Kroesbergen & Van Luit, 2002; Moreno & Durán, 2004; Terwel et al., 2009; Fyfe et al., 2012; Fisher et al., 2013; Carbonneau & Marley, 2015; Fyfe & Rittle-Johnson, 2016) indicated some situations where more guidance was better than less guidance, while one study Sengupta-Irving and Enyedy (2015) found no differences in learning as a function of guidance. However, prior knowledge and special education status appear to interact with level of guidance to produce different outcomes. Fyfe et al. (2012) and Fyfe & Rittle-Johnson (2016) found that students with low prior knowledge benefited more from guidance (high or moderate high) whereas students with more prior knowledge did not. Kroesbergen and Van Luit (2002, 2005) found that students in special education classes benefited more from low guidance than high guidance. Taken together the data indicates some support for the benefits of guidance, but with the caveat that teachers need to consider the special education status level and level of prior knowledge.

Structure Studies

The studies we discuss below implement interventions of various types of structure. As discussed above, we define structure as the purposeful explicitness and organization of the lesson plan, curriculum, or materials for understanding. As stated above, seven studies were found in our search for structure, while five studies were found in our search for guidance and were recategorized as structure based on our definition. These studies are identified with an asterisk. One study by Terwel et al. (2009), found in our guidance search, manipulated both guidance and structure, so we have included this study in our review of structure as well. In all, we found 10 studies that met our criteria for structure. Examples of interventions categorized as structure included worked examples, presentation of formulae or information, or sequential arrangement of problems (e.g., grouping similar problems together). For the discussion below we organized the studies into four types of structure including structure of content for problem solving, structure of sequencing, structure of materials, and structure of learning environments. Some of the studies compared high to low structure but we also reviewed studies that compared different types of structure (e.g., worked examples versus formulas) that do not neatly fit into high and low structure categories.

Structure of content for problem solving. Five studies investigated the explicitness of the structure of content for problem solving. Specifically, the explicitness of information or instructions provided to students for their problem solving. Chen, Kalyuga and Sweller (2015)* compared two forms of structure for learning geometry: providing students with worked examples (high structure) versus independent problem solving (low structure). The worked example effect is improved performance as a function of the organized presentation of problems and scaffolding. The authors compared the effects of worked examples when students solved problems that were high or low in element interactivity, which refers to the working memory load as a function of task demands. To operationalize element interactivity, the authors compared problems they identified as having high element interactivity (e.g., area problems with many elements to be kept in mind at the same time) with problems identified as having low element interactivity (e.g., solving for the area of a problem where the students only needs to remember one element, such as length or width, at a time). Students either received high or low structure instruction with high structure involving worked problems and low structure involving independent problem solving. Overall, the authors found that problem solving with worked examples (high structure) was optimal for problems high in element interactivity as opposed to low element interactivity.

Timmermans, Van Lieshout and Verhoeven (2007)* worked with low-performing fourth grade students on their subtraction problem solving. In their "guided" instruction condition, which we categorized as low structure, students were left to their own devices to develop multiple strategies while solving subtraction problems, i.e., no structure was provided in the form of order or selection of strategies. In the direct instruction condition, which we categorized as high structure, students were trained to use a strategy for solving subtraction problems. Overall, they found no difference in gains from pretest to posttest between conditions. The authors concluded that their guided instruction condition was not a satisfactory alternative to typical direct instruction for teaching low performing students.

As discussed above in the section on guidance, Terwel et al. (2009)* compared the impact of two problem solving lessons on student learning of percentages and graphs. As a part of that study they compared high and low structure. In the first condition, fifth grade students were provided with ready-made, completed representations (high structure). In the second condition students were taught through the process of guided co-construction, where students and teachers created representations of the percentages without the help of ready-made representations (low structure). Controlling for pretests scores, children in the low structure condition performed better on a posttest and transfer test than students in the high structure condition.

Sidney and Alibali (2015) compared the impact of two forms of structure on the instruction of the division of fractions. One high structure condition involved students linking practice on division of fraction problems to previously solved division of fraction problems whereas the low structure condition involved the students just solving the problems. All students learned fraction procedures equally well, but students in the low structure condition performed better than students in the high structure condition on items that assessed conceptual knowledge. The authors proposed that prompting students to link information without providing an explanation as to what they should be linking and why they should be creating the link may have added to cognitive load.

Kaminski and Sloutsky (2013) investigated the complexity of information during instruction specifically in the form of extra, irrelevant information in problems. This study differed from other studies of structure because one condition involved irrelevant information that might confuse students and that required more processing. Hence, the condition with irrelevant information would be considered low structure, as this did not include the purposeful organization of the lesson plan for understanding. Kaminski and Sloutsky compared kindergarten, first, and second grade students' performance reading bar graphs after instruction that included extraneous information (e.g., extra designs within the bars) to no extraneous information. The authors found students who learned without the extra information (high structure) learned more from pretest to posttest compared to students learning with the extra information (low structure).

With the exception of the work using worked examples there is little evidence that structure in the form of explicit problem solving strategies is effective in improving performance. One reason for this is that the addition of structure adds complexity to the problem solving and additional cognitive load. For example, when students are asked to link the current problem to a prior problem or are given completed examples as models this creates additional cognitive load and increases the possibility of confusion. When not combined with guidance, high structure appears to be less effective. Studies where worked problems that were found to be effective provide guidance as well as structure as students are guided through increasingly difficult problems.

Structure as sequencing. Three studies implemented worksheets or problem solving activities in a prescribed order to encourage learning of mathematics rules or shortcuts. In one instance of this, Baroody, Purpura, Eiland and Reid (2014)* compared the effectiveness of three conditions that differed in the order of problem presentation on students' fluency with two rules: subtract-to-add and add-with-10. The first condition, "guided" subtraction, was labeled as high structure because subtraction problems were ordered so that families of addition and subtraction problems were grouped together (e.g., 3+9 and 12-9); the second condition, "guided" use-a-10, was labeled as high structure because it presented problems in a prescribed order designed to show students how to use the add-with-10 strategy. A final third condition was a control group in which problems were not presented in any specified order and was categorized as low structure for that reason. The authors found the subtract-to-add group outperformed both the use-a-10 and control groups on a measure of fluency on unpracticed subtraction problems. The use-a-10 group did not perform significantly better than the control group indicating that the effectiveness of structure may be influenced by the type of rules to be learned.

In a subsequent, similar study, Baroody et al., (2015)* focused on teaching kindergarten through second grade students the add-1 rule (any number plus one is the next number) and the doubles rule (using doubles to compute answers to close numbers). The ordered add-1 rule condition was categorized as high structure because the students were exposed to $n + 1$ problems as a group. The ordered doubles rule condition was categorized as high structure because the doubles problems were presented as a group (e.g., 8 + 8). The control group was categorized as low structure because the problems were randomly presented in no prescribed order. They found students equally learned the add-1 rule in both the ordered and random conditions, but did better learning the doubles rule in the ordered doubles rule condition. This, again, points to the indication that the effectiveness of structure may be influenced by the type of rules to be learned.

Similarly, Purpura, Baroody, Eiland and Reid (2016)* compared the impact of three conditions with at risk first grade students on performance on add-1 problems and doubles problems. The conditions were highly structured add-1, highly structured doubles, or low structure practice. In their two high structure conditions, structure involved the use of a computer program that first highlighted relations (e.g., "What number comes after 5?") followed immediately by an addition problem (e.g., "5+1=?"). In the low structure, practice-only condition, addition problems were ordered randomly with highlighted relations. They found for learning the doubles rule, only the high structure doubles condition was effective. Both conditions (add-1 and doubles) were equally effective for learning the add-1 rule.

The results of three studies point to the effectiveness of sequencing problems in some instances, but not others. The inconsistencies in the findings, however, make it difficult to draw clear conclusions. In some instances, a grouping or sequencing would produce better results. For example, teaching the doubles rule for addition produced better results when students had a sequence of prompts about relationships, but these prompts did not improve the outcome for the simpler add-1 tasks. Future research needs to look more closely at how and when sequencing can support conceptual understanding.

Structure as materials. This section looks at structure in the form of materials used during instruction.These materials might be manipulatives for counting or other materials designed to support counting. It also includes the presentation of representations (e.g., number lines) designed to support counting and learning.

Tournaki, Bae and Kerekes (2008) investigated learning with and without the use of a rekenrek, an instrument similar to an abacus but with a base-five structure instead of a base-ten structure. Forty-five first grade students with mathematics disabilities were randomly assigned to one of two instruction groups, both of which were categorized as high structure, or a third control group (low structure) that received no instruction. Both instruction groups included counting songs, counting activities with manipulatives, counting comparison activities, and fact family activities (e.g., 5 has the fact families 4 and 1 or 2 and 3). The only difference between the instruction groups was the use of the rekrenrek versus the use of fingers only. Students in the rekenrek instruction group performed significantly better than students in the fingers only condition and the control condition on posttest addition and subtraction problems. There was no difference between the fingers only group and the control group. However, the lack of pretest makes it difficult to determine whether the use of rekenrek or pretest differences were the cause of the difference.

Tsang, Blair, Bofferding and Schwartz (2015) compared the relative effectiveness of different types of structure. The authors compared three conditions for fourth grade students counting positive and negative integers on a number line. One condition had students jump a figure along a number line, a second condition had students stack blocks along a number line, and a third condition had students fold the positive and negative sides of a number line together to emphasize cancelling out when problem solving. All three conditions were high structure according to our definition because they provided materials for aiding problem solving. The authors found that students in the folding condition showed evidence of incorporating symmetry into their mental representations of integers and performed higher on transfer tasks compared to students in the other two conditions.

Only one study actually compared low to high structure and that study was flawed by the failure to pretest. The second study did not include a low structure control group but the results indicated that the effectiveness of high structure differed as a function of the materials. The limited results indicate that it should not be assumed that the inclusion of any kinds of concrete materials will consistently improve performance. More research needs to be done on the effectiveness of these materials.

Structure summary. When studies compared high to low structure no clear pattern emerged favoring high or low structure. This was due in part to factors other than structure that appear to interact with structure to affect performance. When high structure was accompanied by high guidance, for example, it was more likely to be effective. Likewise, there was some evidence that structure was more important when there was high cognitive load. These factors need to be considered when selecting a particular type of structure. Highly structured materials or activities that are unfamiliar or that are cognitively demanding may be less effective than less complex materials and activities that are unstructured. Future research needs to focus how cognitive complexity and structure interact. As with guidance it is likely that structure that is complex may be less effective for novices and lower performing students.

Conclusion and the need for future research

Kirschner et al. (2006) review and the commentary articles that followed it highlighted the lack of a clear, consistent definition of guidance and the tendency to confound guidance and structure. The goal of this review was to explore prior research using these two terms in order to create clear, discriminative definitions of guidance and structure. This can allow future research to better understand how guidance and structure impact elementary school mathematics learning. To accomplish this goal, we first differentiated and defined guidance and structure. We then reviewed the literature on guidance and structure using these revised terms to organize our review.

The research on guidance was more homogeneous and more consistent in indicating the effectiveness of guidance. The research also indicated that multiple factors are at play in determining how much guidance to provide to students. For example, when teaching mathematics to students with mild intellectual disabilities, direct instruction with little guidance was found to be more effective than a lesson with more guidance (Kroesbergen & Van Luit, 2005). Likewise, Fyfe et al. (2012, 2015) found that prior knowledge and working memory moderated the impact of high and low guidance on learning. Previous research has shown working memory and prior knowledge influence students' ability to learn during mathematics instruction and problem solving, as a function of cognitive load (Alloway & Gathercole, 2008). Our review indicates that guidance, in general, produces positive outcomes but that teachers need to consider the demands high levels of guidance may place on lower performing students.

The research on structure was more heterogeneous, including a wide variety of features such as worked examples, ordering problems from easy to difficult, or providing formula sheets during problem solving. Furthermore, many of the studies compared different types of structure as opposed to different levels of structure. Our results suggest that structure can improve learning but that complex structure that puts a significant cognitive demand on students is less effective for poorer performing students. This was evident from the different outcomes of different types of structure. Our results indicate that teachers need to consider the nature and quality of structure as much as the amount of structure.

Our review had several limitations. To review guidance and structure with a clear, concise methodology, we greatly limited the studies to search terms that focused on guidance and structure. We limited this review to elementary school mathematics because the effectiveness of guidance may differ as a function of domain and prior knowledge. We also wanted to avoid potential age and domain confounds. Consequently, this limited our ability to find articles that investigated guidance and structure but did not use these terms. There may be more studies that effectively compared different levels of guidance and structure that were not reviewed in this paper. Additionally, we could only review articles that explicitly described the instruction in all conditions. Some articles did not describe instruction in enough detail to determine the level of guidance or structure. However, the articles we reviewed allowed us to determine the inconsistencies in how guidance and structure are defined in order to understand how to best differentiate the two constructs in order to better describe and study them in future studies.

The overarching goal of educational research is to improve student learning. One way this research can affect change is by communicating findings to teachers who then take those findings directly to their classrooms. But in order to provide this information to teachers we must engage in research with clear, specific definitions that can translate to the everyday lessons taking place in classrooms. Prior research related to guidance and structure has provided valuable findings. By creating clear, consistent definitions of these constructs we can accumulate and combine the research that has been conducted thus far in order to better understand this research as well as communicate readily executable finding to teachers.

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