Effects of the association of registers of semiotic representation on the resolution of problems of direct proportionality

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Abstract

This research focuses on the two main registers of semiotic representations in direct proportionality. Based on the theory of registers of semiotic representation, we examine the effects of tabular and graphical registers in solving problems of direct proportionality. We hypothesize that mobilized registers will have an impact on the performance of our learners. Using a factorial design, our study of ninety-two learners in the second year of secondary school (13-14 years old) evaluates the implementation of four pedagogical scenarios characterized by an identical discovery task, but mobilizing different semiotic registers.

Keywords

Semiotic representation registers, proportionality, problem solving, tabular register, graphical register

Résumé

Cette présente recherche s'intéresse aux deux principaux registres de représentations sémiotiques en proportionnalité directe. En nous appuyant sur cette théorie, nous examinons les effets du registre tabulaire et du registre graphique en résolution de problèmes de proportionnalité directe. Nous émettons l'hypothèse que les registres mobilisés auront un impact sur les performances de nos apprenants. À partir d'un plan factoriel, notre étude menée auprès de nonante-deux apprenants *de deuxième année commune du secondaire (13-14 ans) évalue la mise en place de quatre scénarios pédagogiques caractérisés par une tâche de découverte identique, mais mobilisant des registres sémiotiques différents.*

Mots-Clés
Registres de représentations sémiotiques, proportionnalité, résolution de problèmes, registre tabulaire, registre graphique

Introduction

Proportionality has an important place in mathematics. It is not only mobilized in many fields, such as medicine and physics, but also in everyday life. The concept is, for example, used in the dosage of medicines, in calculating the weight (gravitational force) of an object according to its mass, in fuel consumption, in a cooking recipe, etc. The substantial use of this concept therefore gives it a crucial function in teaching (Sokona, 1989). Moreover, it is unavoidable, and a good understanding of it is necessary (Oliveira, 2008) insofar as "…*the ability to master proportional reasoning is a determining factor in the understanding and application of mathematics*" (Ministère de l'Éducation, 2012, p. 4). In spite of its central place as a concept, students have a great deal of difficulty with it (Bertheleu et al., 1997; Comin, 2002; Dupuis & Pluvinage, 1981).

Daro, Géron and Stegen (2007) point out that application exercises are not enough to construct and understand this concept. Instead, they suggest using problem situations. Currently, the guideline in teaching proportionality is to no longer propose resolution strategies for each type of problem, but rather to use these situations to solicit students' analogical reasoning. Moreover, this mathematical object can be accessed through different modalities of representation. In French-speaking Belgian education, the representations envisaged in the second year of secondary education (13-14 years) are graphs, proportionality tables and, occasionally, formulas (Fédération Wallonie Bruxelles, 2013). In considering these different representations, it seems appropriate to us to rely on the didactic notions of registers of semiotic representation (Duval, 1993, 2001, 2007, 2008, 2013a, 2013b). Indeed, these representations are derived from different registers of semiotic representation: the graphical register, the tabular register (associated with the numerical register), and the algebraic register. These registers are usually juxtaposed, and the only inter-register transition envisaged, called "*conversion*" (Duval, 1993, 2007), is from the tabular register to the graphical register. We wonder about the articulation between these two registers, which are the two main registers of semiotic representation in direct proportionality.

Through this research, we study the effects of the association of these two registers in solving problems of direct proportionality. More specifically, we examine the impact of the status of the tabular register (with or without) as a function of the status of the graphical register.

Theoretical framework

Direct proportionality and mathematical benchmarks

Direct proportionality can be defined as "...*a particular relationship between two quantities (or rather their measures) or between two series of numbers. These two series of numbers (with or without associated quantities) must be multiples of each other*" (Daro et al., 2007, p. 20).

Presented under the theory of linear application (Hersant, 2005), this model verifies the multiplicative and additive properties of linearity.

Table 1

Proportionality table representing the price to be paid according to the mass of apples

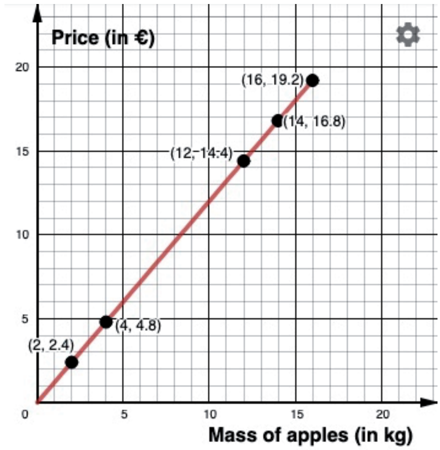
Mass of apples (in kg)	4	2	12	16	14
Price (in €)	4.80	2.40	14.40	19.20	16.80

In this situation (see table 1), the price paid is proportional to the mass of apples purchased. Multiplicative linearity can be defined as follows: if one multiplies (divides) a value of one quantity by a number then one can multiply (divide) the corresponding value of the second quantity by the same number. Concerning additive linearity, it can be stated as follows: if two values of the same magnitude are added (subtracted), then the two corresponding values of the second magnitude can be added (subtracted). This property proves very useful when the proportionality coefficient, obtained by dividing a value of the second quantity by the corresponding value of the first quantity, is too complex, and is probably an approach that should also be proposed to secondary school students (Daro et al., 2007).

This situation of direct proportionality can also be presented using an analytical expression (Price = 1.2 * mass) and a graph (Figure 1).

We can observe that direct proportionality is illustrated by a straight line through the origin of the benchmark (0;0). In this regard, Hersant (2001) speaks of "... *a strong evocative power of proportionality, because it [the line] can be seen at a glance"* (p. 67). Baldy, Durand-Guerrier and Dusseau (2007) say: "*It seems that the drawing of a line passing through the origin is their "prototypical" representation of proportionality*" (p. 207). Furthermore, the authors attribute "this strong evocative power" to the fact that teachers strongly draw students' attention to the graphical representation of a proportionality situation.

FIGURE 1



Graphic representation of this proportionality situation

The illusion of linearity

An excessive use of strategies to solve proportionality problems has been highlighted in non-proportional situations, as identified by De Bock et al. (2007) with pupils in the second year of primary school to the second year of secondary school. The authors attach this behavior to a phenomenon they call the "illusion of linearity", and this systematic recourse is said to be caused by low cognitive investment in problem solving by pupils. These different elements challenge us. According to Gille (2008), the incoherent use of procedures designed to achieve proportionality in non-proportional contexts has its origin in a poor presentation of non-proportional situations to students.

Confronting non-proportionality

In an article explaining the mathematical foundations of proportionality, Simard (2012a) explains that it is necessary for a student to be able to recognize proportionality problems. A student may be able to apply strategies to solve proportionality problems, but may not be able to determine when to use them, as he or she may well use these strategies incorrectly. Thus, the teacher must not only introduce these strategies to students, but also enable them to develop an ability to identify proportionality. Therefore, students should not only be presented with situations of proportionality, but also with situations of non-proportionality. This is also advised by Daro et al. (2007), who even recommend confronting non-proportionality situations as early as possible in order to get students to analyze the statement and avoid this misuse of procedures inappropriate to non-proportionality situations.

Registers of semiotic representation

We can access knowledge objects via two fundamental modes of access, direct (concrete) or indirect (microscopes, telescopes, MRI...) accessibility, and accessibility through semiotic representation, with the first mode being favored. The only exception is in the field of mathematics (Duval, 2008). According to Duval (1993), a mathematical object can only be reached via its semiotic representation. Indeed, mathematical objects are "*... productions constituted by the use of signs belonging to a system of representation that has its own constraints of meaning and function*" (Ibid., p. 39). Moreover, mathematical thought presents, what he calls, a cognitive paradox: "*... on the one hand, the understanding of mathematical objects can only be a conceptual understanding, and on the other hand, it is only through semiotic representations that activity on mathematical objects is possible*" (Ibid., p. 38). The author insists on the importance of the distinction between mathematical objects and their representations in order to avoid "*...a loss of understanding and that the knowledge acquired quickly becomes unusable outside the learning context*" (Ibid., p. 37). Difficulties related to mathematics are said to stem from a neglect of semiosis, "*...the understanding or production of a semiotic representation...*" (Ibid., p. 39) for the benefit of the noesis, "*...the conceptual understanding of an object...*" (Ibid., p. 40).

Cognitive approaches to Semiosis

There are three types of cognitive activities associated with semiosis. First is formation, which is the creation of a representation belonging to a specific register. Second is processing, which is the transformation of one representation into another belonging to the same register as the first. Third is conversion, which is a transformation of a representation belonging to one register into another representation belonging to another register. Processing is a transformation internal to a register while conversion is a transformation external to the starting register. The latter is rarely used in teaching for two reasons. On the one hand, it is judged as acquired by the sole fact of the work done through the formation and processing of representations, and on the other hand, transformation, the result of which is a change of register, would be useless in the understanding of a concept. However, it is precisely this transformation that is crucial in the acquisition of a concept during learning and in noesis (Duval, 1993).

Plurality of registers of semiotic representation

The diversity of registers of semiotic representation is of interest for three reasons (Duval, 1993). First of all, conversions lead to registers in which the processing carried out can be more economical. Second, since each register has its own constraints and possibilities, the resulting representation is cognitively partial, hence the importance of the complementarity of the registers in order to represent the concept in all its cognitive complexity. Thus, the properties of the object are only partially shown by the reductive nature of the registers (Bloch, 2002). Moreover, "... *two representations of the same object in different registers have different contents*" (D'Amore, 2001, p. 159) and these contents are "...*the properties of the object that the register makes accessible*" (Duval, 2001, p. 7). Finally, the formation of mathematical concepts requires the coordination of registers because of the cognitive paradox, and without it, "... *students cannot understand, that is, recognize what is being talked about or take the slightest initiative*" (Duval, 2013a, p. 158). It is therefore necessary to juxtapose several semiotic representations, and this juxtaposition promotes the understanding of the concept if students are able to envisage the conversions that allow them to pass between them (Duval, 2007). Furthermore, Duval (2013a) states that "...*the mathematical solution of a problem requires the use, explicitly or implicitly, of at least two completely different types of representation*" (p. 150). For the author, conversion is considered as the first step of understanding to be overcome at the risk of "...*[causing] a serious handicap in problem solving, which can be observed ... in the application of mathematical knowledge to real-life situations*" (Ibid., p. 152).

Beginning of our experiment

Our observations of the difficulties encountered by students with regard to this concept during our professional practice led us to investigate this concept through our readings on the didactics of mathematics. We quickly noticed the interest of our approach insofar as the complexity of this concept is highlighted by many authors (Comin, 2000, 2002; Daro et al., 2007; Gille, 2008; Hersant, 2001; Lambrecht, 2016; Levain & Vergnaud, 1994; Oliveira, 2005, 2008; Simard, 2012b, 2012a). Given our observations and readings, we wondered about the potential impact of registers of semiotic representation mobilized in proportionality and, more specifically, about their combination (Duval, 1993). Taking into account the different ways in which this concept is represented, and questioning the relationship between them, we opted for research that questions the effect of the association of the tabular and graphical registers, which are the two main registers of semiotic representation in solving problems of direct proportionality. The objective of this experiment was therefore to analyze the effects of the association of the tabular and graphical registers on the resolution of direct proportionality problems.

Research methodology

Experimental design and research sample

Our experimental design takes into account two independent variables: the status of the tabular register and the status of the graphical register. The experimental design we rely on is therefore factorial. The relationship linking our variables is a cross-over relationship. This means that whatever the levels of the first variable and the second variable are, there is a set of observations. We have represented our inter-subject factorial design with two independent variables crossed according to the parallelepiped representation, where the partition criteria of our variables determine four groups of subjects.

Table 2

Sample description

		Status of the tabular register	
		With	Without
Status of the graphical register	With	Group 1 n = 21	Group 2 n = 23
	Without	Group 3 n = 24	Group 4 n = 24

To conduct our research, we selected a qualified sample based on subject availability. Our experimental design required four groups of subjects, so our sample consisted of four second-year high school classes. We randomly assigned the class groups to cross-over modalities of our independent variables. Our sample consisted of 92 subjects.

Research Questions

Based on our experimental design, we formulated three research questions.

Q1) Is the progression of the learners who participated in our pedagogical scenario different according to the status of the tabular register set up? (main effect of the status of the tabular register)

Q2) Is the progression of the learners who have participated in our pedagogical scenario different according to the status of the graphical register set up? (main effect of the status of the graphical register)

Q3) Is there a difference in progression between the learners who participated in our pedagogical scenario according to the status of the tabular register and the status of the graphical register? (interaction effect of the status of the tabular register and that of the graphical register)

In order to assess the individual progress of learners, we calculated the relative gains in each of these situations. These relative gains allow us to measure the ratio between the actual gain (observed progression) and the maximum gain (possible progression). When the first score is less than or equal to the second score, it is a relative gain (Gr) and we have applied the formula: « Score 2 – Score 1 / Maximum – Score 1 » (Relative Gain Formula). In the case where the first score is higher than the second score, it is a relative loss (Pr) and we have applied the formula: « Score 2 – Score 1 / Score 1 » (Relative Loss Formula).

D'Hainaut (1975) considers that from a 30% increase, we can consider that learning has indeed taken place.

Pedagogical scenario

Taking into account the experimental design, we decided to carry out an immediate post-test and a delayed post-test. Our design therefore involves three measurement times: a pre-test, an immediate post-test and a delayed post-test. Below we present the experimental protocol of our research for each 50-minute class session.

TABLE 3

Experimental protocol

Session 1: This session is devoted to the pre-test. In addition to the verification of equivalence between our groups, the students' results made it possible to constitute our trios for the following session.
Session 2: The trios constituted by us on the basis of the individual results carried out the discovery activity. Our treatment took place at this point in the protocol. Although the tasks were identical in the four class groups, the proposed registers of semiotic representations differed from one class group to another.
Session 3: This session is dedicated to the immediate post-test.
Session 4: We read and commented on the synthesis present in our sequence. The first six exercises were then carried out collectively.
Session 5: The last exercises were carried out collectively.
Session 6: This session is dedicated to the delayed post-test.

Our experimental protocol was identical in all respects, with the exception of the second session, which was different in the four groups participating in our experiment.

Measuring instruments: pre-test, immediate post-test and delayed post-test

All our subjects were subjected to an identical pre-test allowing us, on the one hand, to ensure the equivalence between our experimental groups and, on the other hand, to constitute trios of low heterogeneity within each class group. We targeted five main skills and each question in our pre-test was associated with one of these five skills. Based on Régis Gras' taxonomy, reviewed by Bodin in collaboration with Gras (Bodin, 2010), and on the five skills identified, we constructed a test consisting of thirteen questions. We focused on a few sub-categories of the first four hierarchical categories of cognitive complexity of this taxonomy implemented specifically for mathematics. The immediate post-test and delayed post-test items were identical to the pre-test and were placed in the same order. Only the surface data were modified, taking care to keep the same level of complexity both in the nature of the numbers and in their size. The reasoning used, as well as the skills and taxonomic levels, were therefore unchanged.

Constitution of the experimental groups

Since the activity of discovering our scenario requires the manipulation of cylinders and water, it seemed natural to us to form groups of students. First, we determined the size of our groups. Taking into account the number of communication channels and possible inter-individual relationships (De Lièvre, Temperman, & Boumazguida, 2016), the constitution of trios seemed to be a good choice. We formed our trios based on a reasoned method in the hope of "*...creating a dynamic conducive to learning*" (Ibid., p. 73) and by relying on the individual scores of the learners participating in our experiment. In order to do this, we classified the subjects within each group class in ascending order of total score. Second, we divided each group in three to obtain a sub-group of "weaker" students, a sub-group of "average" students and a sub-group of "stronger" students in relation to the assessed direct proportionality skills. For each class, we then randomly selected one student from the first sub-group, one student from the second, and one student from the third. When the number of students was not a multiple of three, we formed a group of four students. In proceeding in this way the heterogeneous groups formed had very similar averages as shown below (Figure 2).

Discovery activity

Our activity was inspired by the publication "Math & manips" of the *Centre de Recherche sur l'Enseignement des Mathématiques* (CREM). We chose our activity for two reasons. Firstly, it leads to a confrontation, recommended in the literature, between direct proportionality and non-proportionality (Daro et al., 2007; Lambrecht, 2016; Simard, 2012a). Second, we believe that it allows the emergence of socio-cognitive conflicts. Indeed, the protocol systematically encourages students to propose an estimate and reach a consensus within the group and then verify it by experimentation. The presence of another individual leads the learner to become aware of the existence of responses different from his own on the same cognitive task. This disagreement resulting from exchanges with others leads to a negotiation between subjects where each person presents his or her arguments (Doise & Mugny, 1997). In the first phase of the activity, the diameter of the cylinder is fixed while the height varies. The objective of this first phase is to get the students to observe that the volume and height of a cylinder are directly proportional quantities (direct proportionality situation). In the second phase, the height of the cylinder is fixed while its diameter varies. This second phase aims to show that the volume and diameter of a cylinder are not directly proportional quantities (non-proportionality situation). Our experimental treatment took place at this point of the pedagogical scenario. In accordance with our experimental plan, we separated the activity into four different forms. Thus, we defined the first form of the activity by mobilizing the tabular and graphical registers, the second form exclusively with the graphical register, the third form exclusively with the tabular register, and the fourth form with the natural language register. We made sure that these four forms were equal in all other respects.

Figure 2

C Selena Rosa	3
W Maya	3
L Margaux	5
F Serena	6
L Sasha	6
M Adriano	7
P Alessia	8
M Thomas	10
U Asya	10
C Elisa	11
D Kenthara	11
E Lorina	11
P Michele	11
D Camille	12
U Romain	12
A Clata	13
P Lola	13
B Priscillia	14
P Alessio	14
L Laura	16
N Ricardo	16
VR Alexis	17
G Yanis	18

M Adriano	7	L Sasha	6	P Alessia	8	W Maya	3
E Lorina	11	D Kenthara	11	C Elisa	11	D Camille	12
P Lola	13	P Alessio	14	A Clata	13	N Ricardo	16
Moyenne	*10,3333*	*Moyenne*	*10,3333*	*Moyenne*	*10,6667*	*Moyenne*	*10,3333*
F Serena	6	G Loic	1	C Selena Rosa	3	L Margaux	5
P Michele	11	U Asya	10	U Romain	12	M Thomas	10
B Priscillia	14	G Yanis	18	VR Alexis	17	L Laura	16
Moyenne	*10,3333*	*Moyenne*	*9,66667*	*Moyenne*	*10,6667*	*Moyenne*	*10,3333*

Constitution of the experimental groups

Activity summary

In the session following the immediate post-test, we presented a summary to the students and commented on it. In accordance with the curriculum, several parts were presented. First, we explained what directly proportional quantities are, then we explained the concept of "coefficient of proportionality" and the formula for obtaining it. We also showed that a situation of direct proportionality can be defined in three forms: a formula of the type y = kx, a proportionality table, and a graph. We also pointed out that the graph of this type of relationship is called a "linear function". Furthermore, we explained the properties of linearity in the context of proportionality tables.

Exercises on direct proportionality

We put together a set of thirteen tasks selected according to Régis Gras' taxonomy, revised by Antoine Bodin (Bodin, 2010), in order to identify their level of complexity and ensure that these tasks covered several levels. The table below shows the ranking of the thirteen tasks according to the five skills being worked on, as well as the associated taxonomic levels. It is important to note that we only targeted the first four categories of the taxonomy, as in the pre-test, immediate post-test and delayed post-test. Tasks 3 and 6 are not associated with any competency. Indeed, task 3 is concerned with the algebraic register and, more precisely, with the algebraic expression reflecting a situation of proportionality (linear function). As this is only officially dealt with in the third year of secondary school, the related competence is therefore not present in the Base (Fédération Wallonie Bruxelles, 2013). As for task 6, it is a question of drawing a graph which depends on a given situation and is not presented as a situation of direct proportionality. In this context, this exercise, which is in line with our discovery activity, is quite rare, but we consider, by its quality, that it deserves its place in our scenario.

For example, Task 1 involves four tables of data related to concrete situations. Students must determine whether they reflect a situation of direct proportionality and justify their responses. If so, the next task is to determine the proportionality coefficient. The aim of this task is to bring out different strategies for deciding on the potential proportionality relationship between two series of numbers, as well as to train students to establish the proportionality coefficient (For more details, see Appendix).

Results

First, we compared the initial abilities of our sample with respect to the concept of proportionality. Since the conditions for the application of normality of distributions and homogeneity of variances were not met, and since the class groups were also of different sizes, we cannot assume that the robustness of the classical ANOVA will certainly compensate for the violation of the assumptions (Cousineau, 2011). For this reason, we used Kruskal Wallis analysis of variance to check the equivalence between our four groups. We can consider that there is no statistically significant difference in the means of the total scores between our four group classes (x^2=3.941; df=3; p=0.268).

We then analyzed the progress of the learners subjected to our pedagogical scenario, both from a descriptive and inferential point of view, by means of a two way analysis of variance (ANOVA) cross-tabulated to the independent groups, the postulates being respected on all of these data. In addition, we distinguished the results obtained from our analyses between our three measurement instruments: pre-test and immediate post-test; immediate post-test and delayed post-test; and pre-test and delayed post-test. The presentation of the results is structured around the three measurement times.

Table 4

Distribution of tasks according to competences and taxonomic categories

		Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Task 7	Task 8	Task 9	Task 10	Task 11	Task 12	Task 13
Skills	Recognize a direct proportionality table among others	×						×						
	In a situation of direct proportionality, complete, construct and exploit a table that relates two quantities	×			×	×		×						
	Solving simple problems of direct proportionality								×	×	×	×	×	
	Recognize and construct enlargements and reductions of figures based on the properties of proportionality.													×
	Interpreting a graph		×											
Categories	A → Knowledge and recognition	×	×	×	×	×								
	B → Understanding				×	×	×							
	C → Application					×		×	×	×				
	D → Creativity										×	×	×	×

Between the pre-test and immediate post-test

TABLE 5

Descriptive analysis of learners' individual progress according to total scores (pre-test and post-test 1)

		Status of the tabular register: With	Status of the tabular register: Without	
Status of the graphical register	With	Group 1 n = 21 ***Average pre-test :*** *40.91%* ***Average post-test 1 :*** *46.32%* ***Average Gr/Pr :*** *7.52%* ***Pre-test CV :*** *59.52%* ***Post-test 1 CV :*** *52.17%*	Group 2 n = 23 ***Average pre-test :*** *50.4%* ***Average post-test 1 :*** *47.83%* ***Average Gr/Pr :*** *-3.23%* ***Pre-test CV :*** *37.67%* ***Post-test 1 CV :*** *46.02%*	n = 44 ***Average pre-test :*** *10.1%* ***Average post-test 1 :*** *10.36%* ***Average Gr/Pr :*** *1.9%*
	Without	Group 3 n = 24 ***Average pre-test :*** *54.36%* ***Average post-test 1 :*** *69.13%* ***Average Gr/Pr :*** *34.2%* ***Pre-test CV :*** *40.18%* ***Post-test 1 CV :*** *31.74%*	Group 4 n = 24 ***Average pre-test :*** *46.97%* ***Average post-test 1 :*** *58.14%* ***Average Gr/Pr :*** *21.62%* ***Pre-test CV :*** *44.82%* ***Post-test 1 CV :*** *40.85%*	n = 48 ***Average pre-test :*** *11.15%* ***Average post-test 1 :*** *14%* ***Average Gr/Pr :*** *27.95%*
		n = 45 ***Average pre-test :*** *10.58%* ***Average post-test 1 :*** *12.87%* ***Average Gr/Pr :*** *21.75%*	n = 47 ***Average pre-test :*** *10.7%* ***Average post-test 1 :*** *11.68%* ***Average Gr/Pr :*** *9.5%*	

Upon observing the table 5, we can see that during the discovery activity only group 2 (which carried out the graphic register task) regressed between the pre-test ($\overline{X}$=50.4%) and the post-test 1 ($\overline{X}$=47.83%). We see an increase from more than 5% to almost 15%, respectively for group 1 (tabular and graphical registers) and group 3 (tabular register). Referring to the average relative gains/relative loss [Average Gr/Pr (in %)], the group 3 which carried out the tabular register activity showed an average relative gain ($\overline{X}_{Gr}$= 34.2%) exceeding 30%, which attests to real learning (D'Hainaut, 1975). Our discovery activity reduced the initial differences between students for all groups as illustrated by the values of the coefficients of variation (CV), except for the second group where the distribution became more heterogeneous ($CV_{pré\text{-}test}$=37.67%; $CV_{post\text{-}test\ 1}$=46.02%). Learners who benefited from the tabular register in the discovery activity show greater progress ($\overline{X}_{Gr}$=21.75%) than those who did not ($\overline{X}_{Gr}$=9.5%). We observed that subjects who did not have the graphical register during the discovery activity made better progress ($\overline{X}_{Gr}$=27.95%) than those who did ($\overline{X}_{Gr}$=1.9%), as shown in the table 5. However, we observe a larger progress gap for the variable "status of the graphical register". This allows us to assume that there would be a significant effect of this variable on the progress of subjects.

TABLE 6

Inferential analysis of individual learner progress according to total scores (pre-test and post-test 1)

ANOVA - Gr pre-post1					
	Sum of Squares	**df**	**Mean Square**	**F**	**p (2 issues)**
Table	3104.07	1	0.024	2.462	0.120
Graph	15260.17	1	0.118	12.106	<0.001
Table * Graph	17.88	1	0	0.014	0.905
Residual	110930.15	88	1260.57		

In the table 6, we analyzed the relative gains between the pre-test and immediate post-test using an ANOVA for two inter-subject factors, namely the status of the tabular register (yes/no) and the status of the graphical register (yes/no). The main effect for the tabular register status was not significant [F(1.88)=2.462; p=0.12]. The interaction effect between the status of the tabular register and the status of the graphical register was also not significant [F(1.88)=0.014; p=0.905]. On the other hand, we identified a significant effect of the status of the graphical register [F(1.88)=12.106; p<0.001].

Between the immediate post-test and delayed post-test

TABLE 7

Descriptive analysis of individual learner progress according to total scores (post-test 1 and post-test 2)

		Status of the tabular register: With	Status of the tabular register: Without	
Status of the graphical register	With	Group 1 n = 21 ***Average post-test 1 :*** *46.32%* ***Average post-test 2 :*** *58.66%* ***Average Gr/Pr :*** *21.59%* ***Post-test 1 CV :*** *59.17%* ***Post-test 2 CV :*** *47.46%*	Group 2 n = 23 ***Average post-test 1 :*** *47.83%* ***Average post-test 2 :*** *62.25%* ***Average Gr/Pr :*** *29.59%* ***Post-test 1 CV :*** *46.02%* ***Post-test 2 CV :*** *36.87%*	n = 44 ***Average post-test 1 :*** *10.36%* ***Average post-test 2 :*** *13.32%* ***Average Gr/Pr :*** *25.77%*
	Without	Group 3 n = 24 ***Average post-test 1 :*** *69.13%* ***Average post-test 2 :*** *74.43%* ***Average Gr/Pr :*** *21.62%* ***Post-test 1 CV :*** *31.74%* ***Post-test 2 CV :*** *25.44%*	Group 4 n = 24 ***Average post-test 1 :*** *58.14%* ***Average post-test 2 :*** *67.23%* ***Average Gr/Pr :*** *28.89%* ***Post-test 1 CV :*** *40.85%* ***Post-test 2 CV :*** *34.07%*	n = 48 ***Average post-test 1 :*** *14%* ***Average post-test 2 :*** *15.58%* ***Average Gr/Pr :*** *25.25%*
		n = 45 ***Average post-test 1 :*** *12.87%* ***Average post-test 2 :*** *14.76%* ***Average Gr/Pr :*** *21.61%*	n = 47 ***Average post-test 1 :*** *11.68%* ***Average post-test 2 :*** *14.26%* ***Average Gr/Pr :*** *29.24%*	

Examination of the table 7 shows that the averages for the delayed post-test are all higher than those for the immediate post-test. As for the average of relative gains, these did not reach 30%, a value from which we can consider real learning (D'Hainaut, 1975).

Nevertheless, the value of the group 2 having benefited exclusively from the graphical register was not far from this value ($\overline{X}_{Gr}$=29.59%). The table also shows a decrease in the coefficients of variation for all the groups, showing a more homogeneous distribution of the results. We identify a slightly higher progression in favour of learners who did not benefit from the tabular register ($\overline{X}_{Gr}$=29.24%). Regarding the status of the graphical register, the progressions observed for each of the modalities are very similar. We therefore believe that these variables do not have a significant influence on the progression of students between the immediate post-test and the delayed post-test.

TABLE 8

Inferential analysis of individual learner progress according to total scores (post-test 1 and post-test 2)

ANOVA - Gr post1-post2					
	Sum of Squares	**df**	**Mean Square**	**F**	**p (2 issues)**
Table	1337.801	1	1337.801	1.159	0.285
Graph	2.650	1	2.650	0.002	0.962
Table * Graph	3.010	1	3.010	0.003	0.959
Residual	101543.181	88	1153.900		

A factorial ANOVA (see table 8) shows that the main effect of the tabular register status is not significant [F(1.88)=1.159; p=0.285], as is the effect of the graphical register status [F(1.88)=0.002; p=0.962]. The interaction effect is also not significant [F(1.88)=0.003; p=0.959]. Therefore, there is no "tabular register status" or "graphical register status" effect on individual learners' progress between the immediate and delayed post-test. Nor can we conclude that there is an interaction effect of the variables "status of tabular register" and "status of graphical register" on these same progressions.

Between the pre-test and delayed post-test

TABLE 9

Descriptive analysis of individual learner progress according to total scores (pre-test and post-test 2)

		Status of the tabular register		
		With	Without	
Status of the graphical register	With	Group 1 n = 21 ***Average pre-test :*** *40.91%* ***Average post-test 2 :*** *58.66%* ***Average Gr/Pr :*** *33.14%* ***Pre-test CV :*** *59.52%* ***Post-test 2 CV :*** *47.76%*	Group 2 n = 23 ***Average pre-test :*** *50.4%* ***Average post-test 2 :*** *62.25%* ***Average Gr/Pr :*** *27.01%* ***Pre-test CV :*** *37.67%* ***Post-test 2 CV :*** *33.87%*	n = 44 ***Average pre-test :*** *10.1%* ***Average post-test 2 :*** *13.32%* ***Average Gr/Pr :*** *29.93%*
	Without	Group 3 n = 24 ***Average pre-test :*** *54.36%* ***Average post-test 2 :*** *74.43%* ***Average Gr/Pr :*** *48.9%* ***Pre-test CV :*** *40.18%* ***Post-test 2 CV :*** *30.44%*	Group 4 n = 24 ***Average pre-test :*** *46.97%* ***Average post-test 2 :*** *67.23%* ***Average Gr/Pr :*** *42.4%* ***Pre-test CV :*** *44.82%* ***Post-test 2 CV :*** *34.07%*	n = 48 ***Average pre-test :*** *11.15%* ***Average post-test 2 :*** *15.58%* ***Average Gr/Pr :*** *45.65%*
		n = 45 ***Average pre-test :*** *10.58%* ***Average post-test 2 :*** *14.76%* ***Average Gr/Pr :*** *41.55%*	n = 47 ***Average pre-test :*** *10.7%* ***Average post-test 2 :*** *14.26%* ***Average Gr/Pr :*** *34.87%*	

This table 9 allows us to observe a positive evolution of the averages between the pre-test and the delayed post-test for all the groups. Regardless of how the registers are recorded, the average relative gains show a progression for the whole sample. The only group displaying an average of relative gains below 30%, the threshold of real learning (D'Hainaut, 1975), is the one that had the activity with only the graphical register ($\overline{X}_{Gr}$=27.01%). The heterogeneity within the groups decreased between the pre-test and the delayed post-test. The progress of subjects who have benefited from the tabular register ($\overline{X}_{Gr}$=41.55%) is higher than that of learners who have not benefited from it ($\overline{X}_{Gr}$=34.87%). As the data in the table 9 show, the difference between the relative gains of learners without and those with the graphical register is large, with a difference of almost 16%. Indeed, the highest increases are in favor of the groups without the graphical register ($\overline{X}_{Gr}$=45.65%). This difference in progression leads us to believe that this variable would have a significant influence on learners' progress.

TABLE 10

Inferential analysis of individual learner progress according to total scores (pre-test and post-test 2)

ANOVA - Gr pre-post2					
	Sum of Squares	**df**	**Mean Square**	**F**	**p (2 issues)**
Table	915.780	1	915.780	0.793	0.376
Graph	5564.570	1	5564.570	4.821	0.031
Table * Graph	0.816	1	0.816	7.065e-4	0.979
Residual	101577.227	88	1154.287		

Similar to the analysis between the pre-test and immediate post-test, the analysis of variance (see table 10) indicates a significant effect with respect to the status of the graphical register [$F(1.88)=4.821$; $p=0.031$]. We did not identify any effect of the "tabular register status" variable on the individual progress of learners, nor any interaction effect between the two variables.

DISCUSSIONS AND PERSPECTIVES

This study examines both the independent effect of tabular and graphical register as well as the combined effect of each factor on learner progression. Specifically, we investigated the effects of combining these two main registers in solving direct proportionality problems. To do so, we developed a pedagogical scenario, using Bodin's taxonomy (2010), working on proportionality skills and tested it with students of the second year of secondary school. Our research hypothesis was that *"the modalities of semiotic representations would impact the performance of the learners subjected to our pedagogical device".*

Although inferential analyses do not allow us to state that the observed differences in progress are statistically significant, we found that learners who benefited from the tabular register progressed more than those who did not between the pre-test and immediate post-test as well as between the pre-test and delayed post-test. Although the table can detach learners from the context of the problem (Oliveira, 2005) and make them focus on certain boxes, distancing them from an overall analysis (Galai et al., 1990), we believe that the tabular register has led to a better understanding of the properties of linearity and proportionality coefficient for learners with it in the discovery activity. These different links prove to be useful in solving situations of proportionality and we hypothesize that this knowledge acquired in a specific context – *in the discovery activity* – was transferred (Tardif, 2004) where the learners reused it in the tasks proposed in the post-tests. This could explain the greater progression between pre-test and immediate post-test for subjects who benefited from this registry, as this effect seems to be confirmed in the long term between pre-test and delayed post-test.

Inferential analyses of learners' progress have enabled us to show that those who benefited from the graphical register in the discovery activity progressed less than learners who did not benefit from it between the pre-test and immediate post-test, and between the pre-test and the delayed post-test. A closer analysis of the progress of each of the four groups indicates that learners who benefited only from the graphic register made the least progress. In our opinion, this register had a deleterious effect on the progress of our learners in our pedagogical sequence. Despite the fact that the line passing through the origin had "*... a strong evocative power of direct proportionality...*" (Hersant, 2001, p. 67) and is its "*... prototypical representation...*" (Baldy et al., 2007, p. 207), we believe that this register offers a more partial representation of the mathematical object under study and has a more reductive character (Bloch, 2002) than the tabular register. This leads us to believe that the contents made accessible by this register (D'Amore, 2001) present less properties of the object studied (Duval, 2001). Furthermore, we hypothesize that this register presents a higher level of abstraction than the tabular register.

As for the interaction of our two independent variables, we observed no inferentially significant effect of the tabular register status depending on the level of the graphical register status, and vice versa. In this context, our observations are not in line with Duval's theory (1993, 2001, 2007, 2013a). Indeed, he indicates that since registers have their own constraints and possibilities, their complementarity is necessary in order to represent the concept in all its cognitive complexity. Moreover, the mathematical object is formed by the coordination of registers, which is indispensable because of the cognitive paradox, in order to represent the mathematical object in all its cognitive complexity. Even though "*...the mathematical solution of a problem requires the use, explicitly or implicitly, of at least two completely different types of representations*" (Duval, 2013a, p.

150), we agree with the words of Géron et al. (2016) who believe "*that problem solving ... requires a good command of the language ... both in reading and comprehension and in writing. This can be a stumbling block for students*" (p. 2). When we took the tests, some students asked us about statements. These questions often stemmed from reading difficulties or poor comprehension. We believe that the first crucial step in solving a problem is reading the statement, which is a real obstacle to overcome for some learners who find themselves helpless in the face of the problem. Therefore, it would be appropriate to start working upstream on the comprehension processes of a mathematical problem with learners (Laflamme, 2009).

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Appendix

Task 1: Four tables of data related to concrete situations are given. Students must determine whether they reflect a situation of direct proportionality and justify their response. If so, the next task is to determine the coefficient of proportionality. The objective of this task is to bring out different strategies for deciding on the potential proportionality relationship between two series of numbers, but also to train students to establish the proportionality coefficient.

Task 2: A graph representing the price to be paid in a cybercafé as a function of connection time is presented and students are asked to say whether the first magnitude is directly proportional to the second, justifying their answer. This task involves the graphical representation of a situation of direct proportionality.

Task 3: Students are required to circle the formulas that describe a situation of direct proportionality.

Task 4: Students are to determine, for three tables, the coefficient of proportionality and then complete them. This task reactivates the strategies identified for Task 1. Indeed, students can use the value of the coefficient or the properties of linearity or a combination of these strategies to complete the tables.

Task 5: In a concrete situation of slope, students must complete the table, calculate the coefficient of proportionality, and then establish the formula to quickly calculate the slope as a function of horizontal distance. This task combines the proportionality table, the coefficient and the formula.

Task 6: Knowing that a liquid occupies a volume of 120 cm^3 for a height of 5 cm in a rectangular parallelepipedic container, students must draw the graph representing the volume as a function of the height of the liquid. The objective of this task is to get students to make the connection with phase 1 of the discovery activity with the cylinder, since the volume and height of a rectangular parallelepiped are directly proportional quantities.

Task 7: Knowing the average length of time it takes to download 3 songs from the Internet, students must complete the table linking the number of songs to the download time. This task is similar to Task 5, except that it requires an additional step in the resolution (conversion of minutes to seconds).

Task 8: In a geometric context, students must determine the height of two pencils based on the height of a given pencil and then draw one of a specific height. The direct proportionality relationship is underlying and students must see it in order to complete this task.

Task 9: Students must calculate the amount to pay to paint a room knowing the area of the wall surface, the yield of the paint and its price. This involves applying proportional reasoning to a problem.

Task 10: Knowing the dimensions of the negative of a photo and the length of the photo, once it has been printed, students are asked to determine the measurement of its width. This task allows students to realize that this is a proportional enlargement.

Task 11: This task involves calculating the volume of water that has escaped in 14 days from a leaking faucet. This is a direct proportionality problem that involves several steps in the reasoning in order to answer the question posed.

Task 12: Learners are asked to determine the price of perfume bottle B knowing the price of bottle A and knowing that the price is directly proportional to the volume. The dimensions of the bottles being given, the volumes can be determined. Again, this is a problem of proportionality, but the students must understand that the volumes of the bottles must be calculated in order to solve this problem.

Task 13: This task is an adaptation of Brousseau's famous puzzle. Students must construct a reduction of the given puzzle as follows: a 5 centimetre segment measures 4 centimetres in the reduced puzzle. The objective is to consider direct proportionality in a geometric context and deconstruct the common idea of removing one centimetre from each segment.