Young children's graphical sign lexicons and the emergence of mathematical symbols

MAULFRY WORTHINGTON

Faculty of Behavioural and
Movement Sciences, Educational Studies
Vrije Universiteit, Amsterdam
The Netherlands
maulfry@blueyonder.co.uk

ABSTRACT

Young children's personal repertoires or lexicons of graphical signs comprise multiple and diverse signs and symbols. These signs support understanding and progress of the symbolic languages of the culturally established, alphanumerical systems, development evolving early in childhood. Investigating language and inscriptional systems - including those that are drawn, written and mathematical - this evidence-based position paper explores the extent to which children's graphical sign lexicons support their emergent understandings, as they move from intuitive marks and informal signs to formal symbols. These inscriptions are indispensable in communicating ideas, and have significance for the study of young children's understanding of the abstract symbolic language of mathematics.

KEYWORDS

Early childhood, graphical sign lexicons, repertoires, children's mathematical graphics, emergent learners

RÉSUMÉ

Les répertoires personnels ou lexiques de signes graphiques des jeunes enfants comprennent des signes et symboles multiples et divers. Ces signes favorisent la compréhension et le progrès des langages symboliques des systèmes alphanumériques culturellement établis, le développement évoluant dès la petite enfance. En étudiant le langage et les systèmes d'inscription - y compris ceux qui sont dessinés, écrits et mathématiques - ce document de synthèse basé sur des données probantes explore la mesure dans laquelle les lexiques de signes graphiques des enfants soutiennent

leur compréhension émergente, alors qu'ils passent de marques intuitives et de signes informels à des symboles formels. Ces inscriptions sont indispensables pour communiquer des idées et ont une signification pour l'étude de la compréhension du langage symbolique abstrait des mathématiques par les jeunes enfants.

Mots-Clés

Petite enfance, lexiques de signes graphiques, répertoires, graphiques mathématiques des enfants, apprenants émergents

Introduction

"Those of us who have devoted out lifetimes attempting to understand the origin and development of expressive, representational and symbolic thought in infancy and childhood, and how best to support it, quickly came to realize that the beginnings of linguistic and mathematical thought are embedded in rather commonplace actions and drawings made by the infant and young child […] developmentally, these beginnings are of the most profound importance" (Matthews, 2006, pp. xiii-xiv). Graphical inscriptions developed in humans' evolutionary drive to communicate. By three to four years of age children have established an evolving lexicon or repertoire of graphical signs, able to express and communicate their thinking through their literacies, drawing, emergent writing, and emergent inscriptions made in contexts that may be considered mathematical. When provided with meaningful opportunities, young children will freely initiate and communicate through graphical inscriptions, extending their existing cultural knowledge in open contexts including pretend play. Building on earlier work by Carruthers and Worthington into children's mathematical graphics (e.g. 2005, 2006), this article augments more recent doctoral research (Worthington, in process) into the genesis and evolution of young children's mathematical inscriptions. It draws on Vygotskian cultural-historical and social-semiotic theories (1978), and coupled with a usage-based view of language acquisition (Langacker, 2008; Tomasello, 2003), it demonstrates the significance of rich repertoires of signs for young children's emergent mathematics.

This article exhibits the extent to which children's graphical sign lexicons support their understandings as they move from informal and intuitive graphical marks and signs, to formal alphanumerical symbols[1]. Graphical signs may be likened to *external representations; inscriptions*; *notations*; *cultural*, *psychological* or *symbolic tools; emergent models;*

1 In Carruthers and Worthington's interpretation, the word *graphics* refers to all aspects of visual representation, including drawing, children's maps, writing and mathematical inscriptions. Anning (2003) and Carruthers & Worthington (e.g. 2006) extend the term *graphics* to *graphicacy*, indicating the ability to understand, employ and generate graphical inscriptions. The etymology of *graphicacy* relates to *literacy* (https://www.merriam-webster.com/dictionary/graphicacy) and *numeracy*.

schematisations; visual signs, and, from Carruthers & Worthington (e.g. 2005, 2006), *children's mathematical graphics*[2]. Johnston (2010) asserts that probably the most significant aspect of development in infancy is the acquisition of spoken language, emphasising that from their earliest babbling and "*without explicit teaching*, toddlers move from hesitant single words to fluent sentences, and from a small vocabulary to one that is growing by six new words a day. New language tools mean new opportunities for social understanding, for learning about the world, and for sharing experiences" (p. 1, emphasis added).

Equally remarkable is children's ability to use graphical signs to signify meanings and communicate ideas. Rieber and Robinson (2004, p. 154) considered development from babbling to speech "a qualitative transformation from one form to another […] sign-using activity in children is neither simply invented nor passed down by adults […] [becoming] one only after a series of qualitative transformations". Athey (2007) notes that infants' babbling has been compared to scribbling. Over time these early scribbles become differentiated, their drawings, writing and mathematical inscriptions emerging "as notational systems that originate from a common core of […] non-representational graphical marks" (Levin & Bus, 2003, p. 892), understanding of the different inscriptional systems developing gradually. Similarities exist too between young children's emergent writing and their emergent mathematical inscriptions (e.g. Carruthers, 1997; Carruthers & Worthington, 2006; Clay, 1975; Ferreiro & Teberosky, 1979; Tolchinsky, 2003). Young children's signs evolve over time into rule-based structures (Langacker, 2008), a process of *grammaticisation*, or *usage-based* theory of language acquisition (Worthington, Dobber, & Van Oers, 2019, p. 93). In keeping with this theory, it is *language use* that facilitates language structure (Tomasello, 2003). Rather than knowledge (of signs) being transmitted from teacher to children, Van Oers (2001, p. 63, emphasis in the original) maintains that children need to be engaged in solving problems with signs they have invented, focusing "above all on the processes of *structuring* instead of the mastery of fixed and prescribed structures".

In a compelling study Cohn (2012) investigated connections between the development of oral language and drawing, maintaining that both involve numerous connecting elements including children's interplay with the drawings of their culture, their personal incentives, and the social interactions in which they engage. The graphical lexicon "must include individual graphemes *that compose the basic graphic parts of a representation* (i.e., dots, lines, curves, circles, squares, etc.)" (p. 173, emphasis added). As Cohn emphasises, "the drawing system is structured like the linguistic system, and,

2 With concerns about its lack of precision, Carruthers and Worthington prefer not to use the generic term *mark-making*. Not only does it lack clarity, but it fails to do justice to young children's powerful thinking and the many ways in which children choose to explore and communicate in their thinking.

thereby, has an analogous development […] similar to language in that it uses a 'lexicon' of schematic models stored in memory that combine generatively to create innumerable novel images" (p. 172).

Cohn's 'basic graphic parts' have also been identified by Worthington (2009), but with children of 3 to 5 years, reflecting Lancaster's (2014) findings with children under three years of age, and indicating the older children's developing sign use. Machón (2013) and Matthews (1999) have also made exhaustive and important studies of children's early drawings. Machón (p. 322) identified a range of signs that "give rise to equivalents [that] are at a midpoint between graphic symbols and writing signs", this period of experimentation and expansion of graphic symbols is, according to Machón "undoubtedly *the most important in the entire graphic development*" (p. 95, emphasis added). As Lancaster observes, "clearly this involves sophisticated analyses and a considerable degree of understanding of the salient features of the different modes of notation [suggesting that] children's learning about domains of symbolic representation is a continuous, developing and expanding process, which starts very early in their lives" (2003, p. 148).

Mathematical inscriptions

Graphical inscriptions are integral to mathematics at all levels of education and in work and society, and for emergent learners understanding develops through all their literacies (e.g. drawing, writing and mathematical). In primary school children will need to use and understand tables, charts, maps, graphs, algebra, diagrams, geometrical shapes and algorithms, all of which require the use of mathematical signs. Awareness deepens through children's social engagement with family members, peers and teachers. In social contexts children draw on their cultural knowledge of home and school, engaging in a form of apprenticeship that includes both infants' language learning (Rogoff, 2003) and children's learning of graphical inscriptions (Van Oers, 2001).

Young children's difficulties with the abstract symbolic language of mathematics were first identified by Ginsburg (1977), and subsequently by Hiebert (1984) and Hughes (1986): however, since these studies were published little seems to have changed to improve young children's experiences of the 'written' language of mathematics, in the UK or globally. Stipek (2013, p. 434) stresses that whereas most experts on the field of early childhood education support the advancement of "profound mathematical understandings […] that children enjoy", although this has not always happened. In early childhood mathematics the dominant world view is of transmission 'skills-based' teaching, children's use of their personal signs seldom considered to be *real* mathematics. Ernest (2018) observes that traditionally, learners must learn to use the language of mathematics "with great precision" contending "the net result of extended exposure to and practice in mathematics is a social training in obedience, an apprenticeship in strict subservience to the text […] Furthermore, the rule following is done *without any need for attention to the meaning of the signs* […] *from a very early age,* [rather than] *relational understanding, which carries in addition knowing how and why*" (pp. 192-193, emphasis added). This alienation has consequences, Ernest stresses, some children experiencing low levels of confidence resulting in mathematical anxiety; negative attitudes to this subject and repeated failures that limit learning, and, as a result become "self-reinforcing and self-perpetuating, a vicious cycle" (p. 203).

Children's mathematical graphics

Since the early 1990s Carruthers and Worthington have researched children's mathematical inscriptions, believing that children should employ their personal ways of representing, and, rather than relying on only a single way of representing provided by their teacher, should have *ownership* of their representations. In accordance with Vygotsky, "writing [and, by association, children's mathematical graphics] must be 'relevant to life' […] meaningful for children, that an intrinsic need should be aroused in them, and that writing should be incorporated into a task that is necessary and relevant for life […] writing should be taught naturally" (1978, p. 118).

Using their own signs and strategies, Carruthers and Worthington found, facilitates children's understanding of graphical sign-use, helping them to close the gap Hughes (1986) identified, between mathematics with concrete resources (or 'practical' maths); and between their informal signs and the abstract signs of the culturally accepted system of 'written' forms of mathematical inscriptions. Using their own signs also increases children's mental repertoire of signs, (which, in any case, cannot be achieved exclusively through direct teaching of graphical signs).

Carruthers and Worthington's approach emphasises the importance of *emergent modelling*, in which the teacher frequently models various ways of representing related to the children's interests and the mathematics they explore[3]. As Tolchinsky (2003, p. 101) emphasises, essentially, the intention is that children consider a range of potential possibilities (Kamii & DeClark, 1985, p. 35), and compare their own perspective with others, "rather than working on fixed symbolic representations or verbal explanations". Emergent modelling is significant in that it introduces new signs to the children's lexicons, and supports "the emergence of formal mathematical knowledge" (Gravemeijer, 1999, p. 175).

Adding to Carruthers and Worthington's research into young children's mathematical graphics, is that by Cook (e.g. 2001), Munn, (e.g. Munn & Schaffer, 1993) and Papan-

---

3 Emergent modelling is also a feature of *Realistic Mathematics Education* in the Netherlands, though Carruthers and Worthington's use of such models was developed independently. Emergent modelling is significant in that it introduces new signs to the children's personal lexicons in contextually meaningful situations.

dreou, (e.g. 2019), Papandreou and Tsiouli (2020, p. 2) observing that research into children's mathematics "as expressed in naturally occurring activities in pre-school settings is not extensive". More recently the author's doctoral research (in process) focuses on the genesis and development of young children's mathematical inscriptions: in this longitudinal, ethnographical study, documentation of 3-4-year-old children's spontaneous pretend play (in which the children were free to explore, collaborate, self-initiate and communicate their thinking), and their graphical inscriptions were analysed[4].

Theoretical framework

Lexicons

Humans develop personal lexicons or repertoires of signs in order to communicate, something that can be readily identified in infants' developing oral language. Nation (2014, p. 1) writes that lexical skills are "a crucial component of language comprehension and production [...] words [graphical signs] are the building blocks of language [...] resulting in a unit of meaning that can be understood and shared between people". The most obvious indication of children's lexical skills, Nation asserts, is the *breadth* of their vocabulary [or their graphical signs].

As they engage with others' inscriptions in meaningful contexts, individuals' lexicons are augmented and enriched (Worthington, in process). Research (e.g. Papandreou, 2020; Worthington & Van Oers, 2016) has revealed the significance of children's 'funds of knowledge' (Moll, Amanti, Neff, & Gonzales, 1992), which are associated with their personal interests (Worthington, 2018). This finding underscores the importance of meaningful opportunities that enable children *to build on* their cultural knowledge, the cultural context of home providing a 'sociomathematical niche' in respect of mathematical knowledge (Kale, Nur, & Aslan, 2018) prior to (and during) attendance in an educational setting.

Greeno and Hall (1997, p. 367) assert that in education, children's activities should encompass "the rich variety of experience and learning made possible through participation in multiple practices of representation". According to Bybee (1998, pp. 430-431) "repetition enables a speaker to produce language in a more efficient way [...] it facilitates the production of more language, more fluently", although it should always be made in meaningful contexts. Examining two-year-olds' oral word lexicons, Hoff and Naigles (2002, p. 423) write that to understand the various features of word meanings, children "require cross-situational information". As with oral language acquisition,

4 In Worthington's research (in process), documented observations also included a small number of 'open' activities in which the children were also free to decide if, and how they might represent their thinking through graphical inscriptions.

children are more likely to be successful in meaningful contexts in which they can engage with others. Developing a rich lexicon requires collaborative engagement and dialogue, and includes imitation, adaptation and emulation of others' graphical communications (Worthington et al., 2019). Accordingly, children benefit from their *intertextual* engagement, suggesting shared communicative culture by 3-4 years of age. Worthington, Dobber & Van Oers (submitted), maintain that the expansion of the children's graphical repertoires through intertextual awareness is beneficial, enabling them "to select what was, for them, the most appropriate sign from one context, to 'fit' in another, and for a particular communicative purpose".

Research into young children's emerging graphical signs

In her intriguing research of how children aged between 18 and 36 months learn various representational systems, Lancaster (2014) writes that, "Far from being part of a complex grammatical system that is closed to the uninitiated, the structuring principles that underpin syntactic structures are accessible to children long before they are fully able to use conventional systems. Indeed, they are used as a means of structuring the texts that they create, and learning about the very systems of which they are part" (p. 44).

Lancaster (2007 p. 139) revealed how very young children exploit various graphical marks, observing, "mark types do not have fixed referents, but can be repeated and used in different environments [contexts], with environments being the significant factor determining meaning […] a graphic mark can be used to represent different systems [e.g. drawing, writing and their mathematical graphics] and objects" (pp. 150-151). Although these early marks do not themselves constitute recognisable systems of writing or mathematics, Lancaster (2003, p. 151) asserts that they possess qualities that may be integrated into these systems, children's early graphic signs as part of "a system which continues to evolve and to remain useful; nothing is wasted […] development, in other words, is continuous".

Young children's graphical inscriptions are multimodal, decisions made concerning their mode, materiality, and affordances (Worthington & Van Oers, 2017) and challenge conventional perceptions of literacies (Kress, 1997). In an example from Lancaster (2014, p. 39), and following a visit to Euro-Disney, Ruby (2 years, 8 months) talked to her mother, making a series of dotted notations that signified specific elements as she recalled the dragon "being there" (as she pointed to one of her dots), then pointing to another dot, "he's got big eyes", and to another, "and he breathed fire [pause] smoke". Carruthers and Worthington (2011) also identified instances of children using dots in drawings and within inscriptions signifying 'writing', some using dots to signify *unspecified* quantities. One child drew numerous dots to denote "*lots* of baddies" (Worthington et al., submitted). Children also sometimes use dots to denote a *specific* quantity, as when one boy drew and counted the many seeds he intended sowing, others using dots to signify items counted or subtracted.

Investigating drawings of three- to five-year-olds' (Worthington, 2009) identified children exploiting other 'basic graphic parts' in their drawings, for instance, using zigzags to signify certain animals (*crocodiles, dragons, monsters*, suggesting ferocity or physical power). Others used zigzags to denote attributes of people or clothing (*hair, teeth, beach shoes*); natural features (*clouds, sky, water, 'prickly bushes')* and forms of power (*lightening, electricity*); zigzags to signify *birds, flags, stairs*, and *caterpillars*. By 3-4 years of age children also often use horizontal zigzag or wavy lines to signify 'writing' (as in figure 1 and 2), either reflecting an idea of the movement or action of the hand writing, or (to them) its visual pattern (e.g. Carruthers & Worthington, 2006, 2011; Tolchinsky, 2003).

Categories of signs

Peirce identified three categories of signs, *iconic*, *symbolic*, and *indexical* (Buchler, 1955). *Iconic signs* have some resemblance to the object signified, the term *symbolic* referring to conventional symbols (e.g. letters, numerals). *Indexical* refers to something directly connected to that which is signified and, in children's graphics, includes arrows. Employing Peirce's semiotic theory to analyse children's signs assisted in identifying and interpreting children's inscriptions (Worthington et al., 2019). However, this analysis necessitated a new code for the children's *early marks, (*or *early mathematical marks)*[5].

The term 'scribbles' is widely used as a negative term, to refer to careless marks[6]. In the context of early childhood, Carruthers and Worthington (2006) employ the term "*scribble-marks*" to refer to marks that adults might find difficult to interpret without assistance from the child. In their pretend play Worthington et al. (2019, p. 99) found that children appeared to sometimes use scribble-marks "as *shorthand* for communicating meanings" (in spite of the fact that they might write standard letter-symbols in other contexts). This suggests that in their pretend play, rapidly made scribble-marks permit the play to continue without interruption. Such signs are referred to by Werner and Kaplan (1963, pp. 42-43) as "*protosymbols*" that directly '*present*' meanings rather than '*represent it*' and are "extremely important in the genetic process of symbolization". Price, Jewitt and Crescenzi (2015, p. 132) deem the 'meaning' of such marks "initially only available to the child [...] as their symbolic understanding progresses [they] become more recognizable to others". The children's examples below are from a nursery school and a reception class in the southwest of England: they are analysed

5 The terms *early marks, (*or *early mathematical marks)* are taken from Carruthers and Worthington (2005, 2006).

6 Scribbles are a fascinating area of research, but regrettably there is insufficient space in this article to explore them in depth.

using the categories discussed, all revealing the 'basic graphic parts' to which Cohn (2012) refers.

FIGURE 1

Spontaneous letter writing

Genre	Sign category	Graphical signs
Letter writing	Early Marks	Scribble-marks
	Iconic	Drawings (self and mummy), Hearts Wavy-line writing Letter-like signs Numeral-like signs Crosses[7]
	Symbolic	Standard letters

Olivia (4 years, 7 months) confidently filled her page with signs and symbols in a left to right orientation, drawing herself and her mother and positioning them centrally.

7 Crosses can be understood as formal symbols when used in calculations. However, in the children's examples here (with the exception of figure 5, Amelie's dice game), they are not.

Beginning with an (almost) letter-writing convention she wrote "ta [to] mummi". Continuing with a combination of signs and symbols, she expressed her feeling for her mother by adding many crosses (for kisses?) and drawing hearts. In covering her whole paper with graphical signs perhaps she recalled how writing and drawing can fill pages of a story book or other printed texts.

FIGURE 2

James's invitation

Genre	Sign category	Graphical signs
Birthday party invitation	Early marks	Scribble-mark
	Iconic	Dots Circles Squares Triangles Cross Ticks (or letter-like signs) Wavy-line writing

James's group were writing birthday party invitations, which would be followed that afternoon by a real party. James (3 years, 11 months) shows his excitement, explaining. "*This is all the things! This is the oven where the cakes are going in and they are already in and the party is starting in a minute*". Then, pointing to the person he'd drawn at the top-right of his paper, James announced "*The people*" (a drawing of one child – economically – standing for several). Van Oers (2005, p. 9) refers to this use of *one for many* as "imagination as etcetera-act […] suggesting more than can actually be seen, and going "beyond the information given" (p. 15); (see also Worthington, 2010, p. 132). James explains the two large circles, (each with a dot in their centre) as "*enormous lights, party lights on the roof*", (referring perhaps to the ceiling). Finally, and using a time he knew, James announced excitedly, "*one o'clock, now!*".

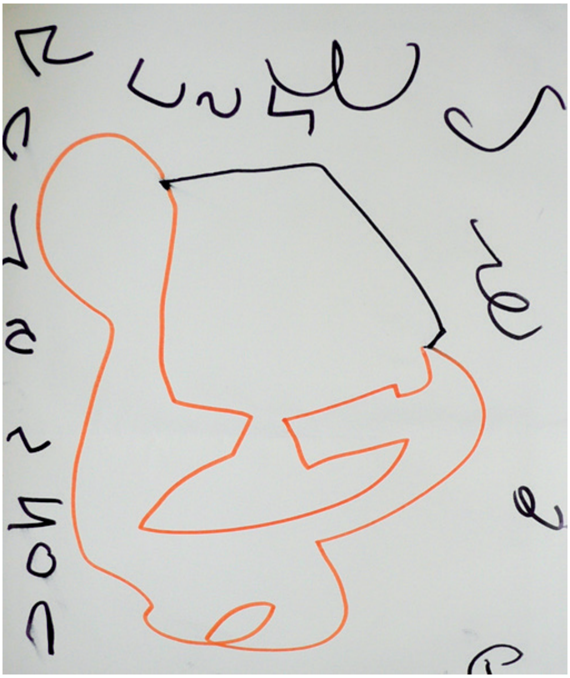
FIGURE 3

"The big bad wolf and the three little pigs"

Genre	**Sign category**	**Graphical signs**
Story telling	Early marks Iconic	Over-drawing Circles, ovals Lines

Jessie (4 years, 1 month) represents her story though drawing, using a range of lines and shapes to describe her thinking. Left of centre she draws the wolf, a tall figure with a tiny head, adding two parallel lines for arms (Figure 3). Her drawing captures the characters in the narrative, to Jessie, the essential elements of her story. For Lancaster (2014, p. 44), young children's spontaneous and freely made signs are "unbounded and flexible".

FIGURE 4



"Sleeping Beauty and the bad witch"

Genre	Sign category	Graphical signs
Story telling	Iconic	Letter-like signs Abstract shapes (straight and curving lines)

Maya's central drawing is non-figurative and assembled of some of the 'basic graphic parts' of which Cohn (2012) writes, the two systems contrasting semiotically. Pointing to her completed drawing Maya (4 years, 4 months) told her teacher "*that's the bad witch*" (Figure 4). Maya understands that children's stories are generally represented by writing and pictures. She made use of her knowledge of writing by using individual, differentiated and separated letter-like signs, fitting them in the space remaining after she had drawn the witch.

FIGURE 5

Amelie's dice game

Genre	Sign category	Graphical signs
Dice game	Early marks	Scribble-marks
	Iconic	Zigzag-line writing Dots Circles Crosses[8] Spiral
	Symbolic	Standard letters and numerals Crosses[9]

8 It appears likely that some of the crosses Amelie included here were signs she knew, but not in an additive context.

9 However, Amelie *also* seemed to have begun to think about using a cross as a symbol for addition, since the children's game involved adding the totals of two dice.

The youngest in her reception class, Amelie (4 years, 4 months) was playing a dice game with friends[10]. Using two dice in each throw she decided to represent their totals each time, making dots on the right of her paper as she counted them out loud (counting one-to-one) (Figure 5). Amelie wrote several capital letter "A's", significant as the first letter of her name. At the top she wrote some numeral-like signs and standard symbolic numerals, enclosing them within circles (copied from other children). In contrast, Amelie wrote several of the numeral "4" (her age) also personally meaningful. She added several other standard letters she knew, and included crosses, these recognised as a "foundational sign" (Worthington et al., submitted). In the centre Amelie has made writing-like wavy marks (unusually, in a vertical, up-down direction), though without giving any explanation of what she intended it to 'say'.

In addition to scribble-marks, Worthington (in process) found that across all their graphical texts, the children used a wide range "basic graphic parts" including dots; lines (straight, curved, parallel, dashed, grids); rectangles (squares, oblongs); circles, concentric circles, arcs and spirals, applying these as elements of their drawings (as in Lancaster's studies), in their maps, writing and mathematical texts[11]. Some children added stars or hearts to drawings they did at home, whilst in their nursery school they also drew arrows, crosses and ticks to convey specific meanings relating to direction, negativity or confirmation. Those with the most extensive graphical sign repertoires, also frequently engaged in drawing at home. These same children used writing-like zigzag or wavy lines; wrote their names most frequently and used the greatest quantity of standard letters, (both upper and lower-case) and standard numerals[12], [13].

Worthington (in process) found that those children having the most extensive 'vocabularies' of graphical signs, also made use of a significant number of formal alpha-numerical symbols, these children appearing to be an advantage regarding their future success in mathematics. For example, Thomas, Mulligan and Goldin (2002) write that it seems that the attentive processing of graphical images has an important role in children's developing understanding of numerals, counting and calculation. Moreover, Merkley and Ansari (2016) and others established that young children's knowledge of symbolic numbers is predictive of ensuing achievement in mathematics, although Worthington cautions against direct and narrow formal teaching of standard numerals. In her research, Papandreou (2019) investigated children's mathematical graphics, analysing four to six-year-old's data investigations and their own strategies as they employed a combination of writing, numerals, calculations or other formal 'written' mathematics.

10 The children in this class were aged 4-6 years.

11 The children in Worthington's study were 3 – 4-year-olds.

12 Other children also used some of these signs, but to a lesser extent.

13 It is interesting to note that none of the children used wavy lines or zigzags to convey *numerical* or *quantitative* information.

As Papandreou observes, her study highlighted the extent to which encouraging children to use and evaluate their inscriptions "may increasingly contribute to an 'emerging metaknowledge' about inscription (Lehrer & Lesh, 2003, p. 369)" (p. 3).

Discussion

Young children are unlikely to achieve fluency and flexibility in sign-use through individual work in traditional, transmission contexts, Carruthers and Worthington's (e.g. 2006) and Worthington's research (in process) accentuates the importance of free and spontaneous pretend play for children's mathematical communications, enabling them to link their existing cultural knowledge to their impromptu mathematical inscriptions.

Blinkoff and Hirsh-Pasek (2019, p. 14) refer to limitations that may restrict children's sign repertoires as "lean language environments". Nation (2014, p. 1), emphasising that, since words are a crucial component of comprehension, it is unsurprising to find children who struggle with language during development: this appears true also for all graphical sign-use. This observation is significant for young learners, for whom their own mathematical inscriptions are never recognised as social or having communicative purposes, and suggest that the difficulties Hughes (1986) and others identified may relate - at least in part - to constraints on this significant aspect of mathematics. Limitations on the breadth of children's graphical sign repertoires suggests limitations on their ability to engage in graphical communication with confidence and fluency, and are likely to impact their subsequent confidence in, and success with mathematics. If children are *only given* the signs and symbols of mathematics, *they will adopt only superficial features*, "unable to transform them into a personally meaningful system" (Ernest, 2005, p. 25).

Brandt & Chernoff (2014, p. 31) write of divisions in mathematics education along ideological lines "of those who perpetuate the idea of a single dominant world view and those who support and cherish diversity". The continuing 'schoolification' of 'official' expectations influences pedagogy, promoting narrow approaches to teaching in reception classes (age 4-5 years), marginalising play, and exerting 'top-down' pressures on preschool and nursery teachers. However, Emfinger (2009) cautions that many teachers forfeit time for play, since they are unable to explain the presence of particular numerate behaviours that pretend play supports. From her research into the pedagogy of children's mathematical graphics, Carruthers (in process) established that the most significant issue cited by most of the nursery teachers in her research (and especially by the reception teachers), are Ofsted inspections, which *considerably impact* on their understandings of play[14]. Raising concerns regarding the amount of control wielded

14 In England children generally start school at 4 years of age and in the 'reception' class are of 4-5 years of age. Confusingly, the curriculum for this age group is that for the early years (from birth

by Ofsted, Carruthers emphasises that Ofsted inspectors' influence is imbalanced and certainly not contested, continuing to be the greatest issue influencing early education in England. Whereas governments have previously given some positive curriculum guidance for early childhood education, Carruthers argues lack of understanding by Ofsted inspectors has culminated in the narrowing of teachers' professional understandings, to which the teachers in her research referred[15].

A recent review of evidence relating to early childhood education in England that includes mathematics (Pascal, Bertram & Rouse, 2019), highlights Carruthers and Worthington's curricula concept (2005, 2011) of children's mathematical graphics, and endorses their approach[16]. Seen through a lexical lens, the examples from these several studies and the research discussed, highlight the importance of freedom for children to cultivate rich repertoires of graphical signs across all their 'literacies' as they begin to employ standard mathematical symbols. Equally important is that teachers and other early childhood professionals should listen, take notice of, understand and sensitively support emergent learners' early beginnings and development of graphical signs (Carruthers, 2015). However, In England the likelihood that children are able to freely represent and communicate their thinking in their own ways feels evermore distant, and Hughes's (1986) concerns regarding young children's difficulties in learning mathematics evermore real.

Acknowledgements

With sincere thanks to the children whose graphics are included in this article, and to their families, headteachers and teachers.

References

Athey, C. (2007). *Extending thought in young children: A parent-teacher partnership*. London, England: Sage.

Anning, A. (2003). Pathways to the graphicacy club: The crossroad of home and pre-school. *Journal of Early Childhood Literacy, 3*(1), 5-35.

Blinkoff, E., & Hirsh-Pasek, K. (2019). Supporting language in the home. *International Journal of Birth and Parent Education, 6*(4), 13-15.

---

to 5 years), and it is because they are in school, that the 'top-down' pressure of the primary curriculum negatively influences children's experiences in reception classes.

15 This lack of understanding appears to be particularly acute in relation to play and children's graphics, but could, Carruthers suggests, be resolved with further training for Ofsted inspectors.

16 This is a collaborative document, with contributions from 12 professional early childhood organisations in England.

Brandt, A., & Chernoff, E. J. (2014). The importance of ethnomathematics in the math class. *Ohio Journal of School Mathematics, 71*, 31-36.

Buchler, J. (Ed.). (1955). *Philosophical writings of Peirce*. London: Dover Publications.

Bybee, J. (1998). The emergent lexicon. *Chicago Linguistic Society 34*, 421-435.

Carruthers, E. (1997). *Number: A developmental theory; a case study of a child from 20 to 44 months*. Masters Thesis, University of Plymouth, England.

Carruthers, E. (2015). Listening to children's mathematics in school. In R. Perry, A. MacDonald & A. Gervasoni (Eds.), *Mathematics and transition to school: International perspectives* (pp. 313-330). Sydney, Australia: Springer.

Carruthers, E. (in process). *The pedagogy of children's mathematical graphics*. Doctoral dissertation, University of Bristol, Bristol, England.

Carruthers, E., & Worthington, M. (2005). Making sense of mathematical graphics: The development of understanding abstract symbolism. *European Early Childhood Education Research Journal, 13*(1), 57-79.

Carruthers, E., & Worthington, M. (2006). *Children's mathematics: Making marks, making meaning*. London, England: Sage.

Carruthers, E., & Worthington, M. (2011). *Understanding children's mathematical graphics: Beginnings in play*. Maidenhead, England: Open University Press/McGraw Hill Education.

Clay, M. (1975). *What did I write?* London, England: Heinemann.

Cohn, N. (2012). Explaining 'I can't draw': Parallels between the structure and development of language and drawing. *Human Development, 55*(4), 167-192.

Cook, D. (2001). 'You can't have a cake unless it's written down': Semiotic activity and authentic learning in play as a potential tool for analysis, *Early Child Development and Care, 168*(1), 49-62.

Emfinger, K. (2009). Numerical conceptions reflected during mutilage child-initiated pretend play. *Journal of Instructional Psychology, 36*(4), 326-334.

Ernest, P. (2005). Activity and creativity in the semiotics of learning mathematics. In M. Hoffmann, J. Lenhard & F. Seeger (Eds.), *Activity and sign: Grounding mathematics education* (pp. 23-34). London, UK: Springer.

Ernest, P. (2018). The ethics of mathematics: Is mathematics harmful? In P. Ernest (Ed.), *The philosophy of mathematics education today* (pp. 187-216). Switzerland: Cham.

Ferreiro, E., & Teberosky, A. (1979). *Literacy before schooling*. London, England: Heinemann Educational.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning, 1*(2), 155-177.

Ginsburg, H. (1977). *Children's arithmetic: The learning process*. Oxford, England: Van Nostrand.

Greeno, J. G., & Hall, R. P. (1997). Practicing representation: Learning with and about representational forms. *The Phi Delta Kappan, 78*(5), 361-367.

Hiebert, J. (1984). Children's mathematics learning: The struggle to link form and understanding. *The Elementary School Journal, 8*(5), 497-513.

Hoff, E., & Naigles, L. (2002). How children use input to acquire a lexicon. *Child Development, 73*(2), 418-433.

Hughes, M. (1986). *Children and number: Difficulties in learning mathematics*. Buckingham, England: Open University Press.

Johnston, J. (2010). Factors that influence language development. In R. E. Tremblay, M. Boivin & R.

De V. Peters. (Eds.), *Encyclopedia on Early Childhood Development* [online] (pp. 1-6). Montreal, Quebec: Centre of Excellence for Early Childhood Development and Strategic Knowledge.

Kale, M., Nur, I., & Aslan, D. (2018). Theoretical framework to examining mathematical experiences in early childhood: Sociomathematical niche. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 12*(2), 1-30.

Kamii, C., & DeClark, G. (1985). *Young children reinvent arithmetic: Implications of Piaget's theory*. New York: Teachers College.

Kress, G. (1997). *Before writing: Rethinking the paths to literacy*. London, England: Routledge.

Lancaster, L. (2003). Moving into literacy: How it all begins. In N. Hall, J. Larson & J. Marsh (Eds.), *Handbook of early childhood Literacy* (pp. 145-153). London, England: Sage.

Lancaster, L. (2007). Representing the ways of the world: How children under three start to use syntax in graphic signs. *Journal of Early Childhood Literacy*, 7(2), 123-154.

Lancaster, L. (2014). The emergence of symbolic principles: The distribution of mind in early sign making. *Biosemiotics,* 7(1), 29-47.

Langacker, R. (2008). *Cognitive grammar: A basic introduction*. Oxford, England: Oxford University Press.

Lehrer, R., & Lesh, R. (2003). Mathematical learning. In W. M. Reynolds & G. E. Miller (Ed.), *Handbook of Psychology*, Volume 7, Educational Psychology, (pp. 357-390). New Jersey: Wiley.

Levin, I., & Bus, A. G. (2003). How is emergent writing based on drawing? Analyses of children's products and their sorting by children and mothers. *Developmental Psychology, 39*(5), 891-905.

Machón, A. (2013). *Children's drawings: The genesis and nature of graphic representation*. Madrid, Spain: Fibulas Publishers.

Matthews, J. (1999). *The art of childhood and adolescence: The construction of meaning*. London, England: Farmer.

Matthews, J. (2006). Foreword. In E. Carruthers & M. Worthington (Eds.), *Children's mathematics: Making marks, making meaning* (pp. xiii-xiv). London, England: Sage.

Merkley, R., & Ansari, D. (2016). Why numerical symbols count in the development of mathematical skills: Evidence from brain and behaviour. *Current Opinion in Behavioural Science, 10,* 14-20.

Moll, C., Amanti, C., Neff, D., & Gonzalez, N. (1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Theory into Practice, 31*(2*),* 132-141.

Munn, P., & Schaffer, H. R. (1993). Literacy and numeracy events in social interactive contexts. *International Journal of Early Years Education, 1*(3), 61-80.

Nation, K. (2014). Lexical learning and lexical processing in children with developmental language impairments. *Philosophical Transactions of the Royal Society, B*, 369, 20120387.

Papandreou, M. (2019). Young children's representational practices in the context of self-initiated data investigations. *Early Years: An International Journal of Research and Development. Retrieved from* https://www.tandfonline.com/doi/full/10.1080/09575146.2019. *1703101*.

Papandreou, M., & Tsiouli, M. (2020). Noticing and understanding children's everyday mathematics during play in early childhood classrooms. *International Journal of Early Years Education.* Retrieved from https://www.tandfonline.com/doi/full/10.1080/09669760.2020.1742673.

Pascal, C., Bertram, T., & Rouse, L. (2019). *Getting it right in the foundation stage: A review of the evidence*. St Albans, England: Early Education.

Price, S., Jewitt, C., & Crescenci, L. (2015). The role of iPads in pre-school children's mark making development. *Computers in Education, 87*, 131-141.

Rieber, R., & Robinson, D. (Eds.). (2004). *The essential Vygotsky.* New York: Kluwer Academic/ Plenum.

Rogoff, B. (2003). *The cultural nature of human development.* New York: Oxford University Press.

Stipek, D. (2013). Mathematics in early childhood education: Revolution or evolution? *Early Education & Development, 24*(4), 431-435.

Thomas, N. D., Mulligan, J. T., & Goldin, G. A. (2002). Children's representation and structural development of the counting sequence 1-100. *Journal of Mathematical Behaviour, 21*, 117-133.

Tolchinsky, L. (2003). *The cradle of culture and what children know about writing and numbers before being taught*. Mahwah, New Jersey: Lawrence Erlbaum Associates.

Tomasello, M. (2003). *Constructing a language: A usage-based theory of language acquisition*. Cambridge, MA: Harvard University Press.

Van Oers, B. (2001). Educational forms of initiation in mathematical culture. *Educational Studies in Mathematics*, *46*(1-3), 59-85.

Van Oers, B. (2005). The potentials of imagination. *Inquiry: Critical thinking across the disciplines, 24*(4), 5-17.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge, Mass: Harvard University Press.

Werner, H., & Kaplan, B. (1963). *Symbol formation. An organismic-developmental approach to the psychology of language.* London, UK: Lawrence Erlbaum Associates.

Worthington, M. (2009). Fish in the water of culture: Signs and symbols in young children's drawing. *Psychology of Education Review, 33*(1), 37-46.

Worthington, M. (2010). Play is a complex landscape: Imagination and symbolic meanings. In P. Broadhead, J. Howard & E. Wood (Eds.), *Play and learning in the Early Years* (pp. 127-144). London, England: Sage.

Worthington, M. (2018). Funds of knowledge: Children's cultural ways of knowing mathematics. In M.-Y. Lai, T. Muir & V. Kinnear (Eds.), *Forging connections in early mathematics teaching and learning* (pp. 239-258). Singapore: Springer Nature.

Worthington, M. (in process). *The emergence and development of young children's mathematical inscriptions: A natural history of signs.* Doctoral Thesis, VU University, Amsterdam.

Worthington, M., & Van Oers, B. (2016). Pretend play and the cultural foundations of mathematics. *European Early Childhood Education Research Journal, 24*(1), 51-66.

Worthington, M., & Van Oers, B. (2017). Children's social literacies: Meaning making and the emergence of graphical signs and texts in pretence. *Journal of Early Childhood Literacy, 17*(2), 147-175.

Worthington, M., Dobber, M., & Van Oers, B. (2019). The development of mathematical abstraction in the nursery. *Educational Studies in Mathematics, 102,* 91-110.

Worthington, M., Dobber, M., & Van Oers, B. (submitted). Intertextuality in young children's inscriptions and their transformations into mathematical symbols. *Mathematical Thinking and Learning.*