# Mathematical teaching in Nursery Schools in England; A way forward for mathematical pretend play and democratic pedagogies

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## Abstract

*Only a few studies have focused on mathematics and pretend play, and they have mostly concentrated on the children and their play and not the supporting pedagogy. To move the debate forward, I present research focusing on the pedagogy of mathematical pretend play, within a socio-cultural perspective, centring on two nursery school teachers, from English Nursery Schools. It revealed the unconventional teaching these teachers engaged in to enhance children's mathematical thinking in play. This may offer us a way into understanding pretend play pedagogies and the conditions that might support children's own mathematical thinking, especially in early years classrooms.*

## Keywords

*Pretend play, pedagogy, mathematics, English Nursery Schools, early years*

## Résumé

*Quelques études seulement ont porté sur les mathématiques et le jeu de rôle, et elles se sont surtout concentrées sur les enfants et leur jeu, et non sur la pédagogie de soutien. Pour faire avancer le débat, je présente une recherche axée sur la pédagogie du jeu de simulation mathématique, dans une perspective socioculturelle, centrée sur deux enseignants de maternelle, issus d'écoles maternelles anglaises. Ces recherches ont révélé l'enseignement non conventionnel que ces enseignants dispensent pour améliorer la réflexion mathématique des enfants dans le jeu. Cela pourrait nous permettre de comprendre les pédagogies du jeu de simulation et les conditions qui pourraient soutenir la réflexion mathématique des enfants, en particulier dans les classes des premières années.*

## Mots-Clés

*Jeu de rôle, pédagogie, mathématiques, écoles maternelles anglaises, petite enfance*

## Theoretical context

### *Introduction*

For the western early years' professional community play is deemed central to children's learning (Siraj-Blatchford & Sylva, 2004) but the pedagogy and understanding of play is complex (Rogers, 2011). How much the adults are involved in children's play is a highly debatable issue. Pramling et al. (2019) discusses that many teachers and practitioners advocate the important emphasis should be on setting up the environment and then the children can freely play with selected resources. Bruce (1991) agrees with a mostly non-interventionist approach which lets the children wallow in their own play worlds. Broström (2017) explains Bruce's concept of play, where children lead, is claimed to come from a Frobelian perspective, however, he argues, this is not in line with Froebel's original concept, "where play can also be adult-directed with a learning perspective. For example, songs and toy materials were incorporated in purposeful play" (Broström, 2017, p. 3). However, the term "purposeful play" which is also used in the Early Years Foundation Stage Statutory Framework in England (Department for Education - DfE, 2017, p. 9) also brings tensions. It gives the impression that play, devoid of careful planning and expected aims, such as children's spontaneous play, is not acceptable. As Wood states, "The emphasis on 'purposeful play' carries the opposite assumption that without pedagogical framing, play would be purposeless" (2010, p. 18). Wood further stresses, free play with few restrictions has always been a problem in educational settings "as it seems to conflict with the set pedagogies of policy frameworks" (2010, p. 18). The pairing of play and pedagogy seems problematic, however, more recent work by Pramling et al. (2019) in an effort to move away from the constant debate on the position of the adult within play, conceptualises a synthesis of play and learning where adults respond to children's play actions without compromising the children's intentions and play narratives.

### *Pretend play*

Few studies have concentrated on mathematical pretend play although the studies that do are not always in child-led activity (Van Oers, 1996; Williams, 2012) which brings somewhat of a conundrum to the research area of play and mathematics, if researchers observe children's play in studies then a prevailing argument is, it should be children who are leading or, at least, the play themes should evolve from children's ideas (Bruce, 1991; Rogers & Evans, 2008). Vygotsky identified pretend play as the leading activity

of young children's learning (1978). An important aspect of pretend play is children use substitute objects and gestures for their meanings in play. This is put forward as contributing to the development of abstract thinking (Bruner, 1996; Vygotsky, 1978) which is vital to mathematical development. Pretend play, in Vygotsky's view, gives opportunities for the development of everyday concepts and provides a channel between spontaneous and scientific concepts (1978, p. 238). Vygotsky saw imaginative and symbolic play as a medium that children can represent the meanings of their individual every day real -life experiences (Vygotsky, 1978). This concurs with research on children's *funds of knowledge* and play as a place where children can own and act out their knowledge (Gonzalez, Moll, & Amanti, 2005). Often the breadth of the zone of proximal development is not considered (Holzman, 1997; Van der Veer & Valsiner, 1991). Play has a learning capacity only when the play environment has the capability to challenge children to span their zone of proximal development. This demands "social interaction where the preschool teacher plays an active role, challenging the child and encouraging him or her to create new meanings and understandings" (Van der Veer 1991, p. 36).

Van Oers (1996) also puts significant weight on children's pretend play, which is often described as imaginary play or role play. Researching mathematical pretend play Van Oers gave, what he termed, an example of good teaching practice, it was of an adult-initiated and led shoe shop role-play. However, this type of role play, although having merits (Van Oers, 1996) restricts the children's scope for imagination as there is too much adult influence (Broadhead, 2010). Adult driven, themed role play is very popular (Rogers, 2010), it is supported by English government play guidance (Office for Standards in Education, 2015) where examples of adult set-up role-play are presented as exemplary practice. This heavily adult-sculpted role play does not necessarily give the children opportunity to pretend or make substitutions which are needed in their play, at the time. This might be because the real things are all there, perfectly supplied by the adults. It is this kind of play which could be said to be distant from the Vygotskian play that is significant for abstract thinking in young children. Perhaps the problem is spontaneous play is sometimes fleeting and catching the learning and the teaching is not always possible for researchers as it is not in a timed lesson; it can happen at any time within the play of the day and it is not known when the most fruitful learning may occur. Interestingly, Broadhead (2010) in an attempt to counter the sterile adult-led and constructed role play areas she had encountered and capture children's own imaginary play recommended the teachers, in their study, set up a "the whatever you want it to be place" (p. 44). The area had ordinary equipment such as cardboard boxes, ropes, tarps and tubing for children to use in any way they needed for their play. A main conclusion, from this research, is when children choose their own play themes and interests, in a mostly unrestricted environment, with an assortment of less traditional

equipment then their pretend play can become more challenging and fulfilling than anything that is adult-led or suggested.

### *Mathematics and play pedagogy*

Ginsburg, in researching mathematical play in pre-school classrooms, agrees that children do acquire a significant amount of mathematics in their own play although he say it is not sufficient, "it does not usually help children to mathematise which means to interpret their experiences in mathematical form and understand the relations between the two" (Ginsburg, 2006, p. 149). In contrast, more recent research findings (Worthington & Van Oers, 2016) observing children's mathematical signs and meanings found children do mathematise in pretend play. Ginsburg promotes playful teaching and gives examples of teachers directly teaching in what he describes as a playful manner. I argue playful teaching and children's own free play are different (Carruthers & Worthington, 2011) as Bodrova and Leong state "adding playful elements to a lesson will not turn it into play" (2015, p. 386). Ginsburg's seeming confusion appears similar to many researchers and mathematical researchers who are seeking answers to a more child appropriate mathematics play pedagogy (Gifford, 2005; Williams, 2012).

However, from a different paradigm, within a socio-cultural lens, recent studies have highlighted the mathematical learning in children's pretend play. For example, Papandreou and Tsiouli (2020) observed children's mathematics in free play, in early childhood classrooms, "to investigate the content, the processes and the origin of children's mathematical knowledge in naturally occurring activities" (p. 1). They highlighted the meaningful mathematical play that children engaged in within a school context. An important part of the research was it centred on children's cultural knowledge which they bring to school and, similar to Worthington and Van Oers (2016, 2017) research, this was highly acknowledged as a useful factor in developing the children's mathematical understanding in play. Worthington and Van Oers (2017) noted the children engaged in many self-initiated literacy opportunities including mathematical signs and symbols. Significantly, this study emphasised, there is a "compelling case for greater appreciation of pretence as a potentially valuable context for the enculturation of literacies" (p. 147) and this includes mathematical literacy. However, the conditions and the pedagogy must be conducive to revealing such rich data and Worthington and Van Oers stated it took them three years to find such practice.

### *Summary*

In summary, pretend play and children's mathematics are not well understood. Often mathematics researchers look for the mathematics which children use in adult set-up role play spaces (Gifford, 2005; Van Oers, 1996; Williams, 2012) which lack the spontaneity and creativity of children's own imaginary worlds or the mathematics

observed is seen as not sufficient (Ginsburg, 2006). The problem is although it has been identified that children, if given the opportunity, use pretend play as a vehicle for rich mathematical learning (Worthington & Van Oers, 2016, 2017; Papandreou & Tsiouli, 2020) it is not common to observe it, or see teachers supporting pretend play, in early years settings (Rogers, 2010). Studies focus on the children and not the supporting pedagogy which I argue is critical for enhancing knowledge and informing practice. I have not found any studies to date which concentrate on the pedagogy of mathematical pretend play. Therefore, using a socio-cultural perspective, this study explores the pedagogy surrounding children's own ways of using and representing mathematics in pretend play situations.

### *Background to the study*

I report on the second part of a research study on teachers' perspectives on children's mathematics including their own mathematical representations. The first part of the study uncovered the differing understandings of play practices in England of seven reception (4 and 5 year olds) teachers and eight nursery school (3 and 4 year olds) teachers. In contrast to the lack of play opportunities in the reception classes, the data revealed all the nursery school teachers put play as a central vehicle of learning. Although they all had differing understandings and emphasis on play, seven out of eight easily produced written observations of children's mathematical play when asked about their mathematical practice. Therefore, an analysis of the nursery teachers' play practice might be helpful for the reception teachers and others who want to enhance mathematical play environments and understand salient aspects of pedagogy.

*The main research question*: What aspects of nursery school pretend play practice can be identified that are useful for supporting children's mathematical perspectives and learning trajectories?

## Methods

This is a qualitative study situated within an interpretativist, post-modern paradigm (Dhalberg, Moss, & Pence, 2006) of listening to children and the rights of children to express their own ways of understanding mathematics in their worlds (Ernest, 2016). It is from a socio-cultural perspective (Vygotsky, 1978) where learning is seen as culturally and socially constructed. I draw on case study research, "to discover patterns and processes to extract lessons learned" (Yin, 2018, p. 105). Although the findings cannot be generalised they may offer a starting point for other researchers or provide a useful comparison for similar research (Atkins & Wallace, 2012)

### *Participants and data collection*

I have selected two vignettes of nursery school mathematics pretend play practice from two nursery teachers who work in inner-city nursery schools. My selection was based on examples of practice which showed sustained and typical practice from the whole nursery school group. The data came from the teachers' written observations and reflections of mathematical practice within their nursery schools in response to a question asking them about their present pedagogical practice in mathematics. The teachers were part of a master's module learning community (Wenger, 2004) which reflected, discussed and debated mathematical practice. The teachers engaged in a synthesis of theory and practice and this praxis (Pascal & Betram, 2012) and the literature they read became a source of awareness and self-reflection (Carruthers, 2015). They wrote notes of the play as they observed it and then reflected on the vignette afterwards in written comments. In the analysis of the data I used sampling (Gilbert, 2004) where new categories are produced and relationships between categories are identified. Secondly, I used conceptual coding looking at lower-level and higher-level concepts and categorised them (Corbin & Strauss, 2015). I present the data in themes and I analyse the data within the themes.

### *Ethical considerations*

Ethical approval for the study was gained from the University of Bristol Ethics Committee. Written consent was given by all participants; pseudonyms are used to protect individual identities.

### *The Vignettes*

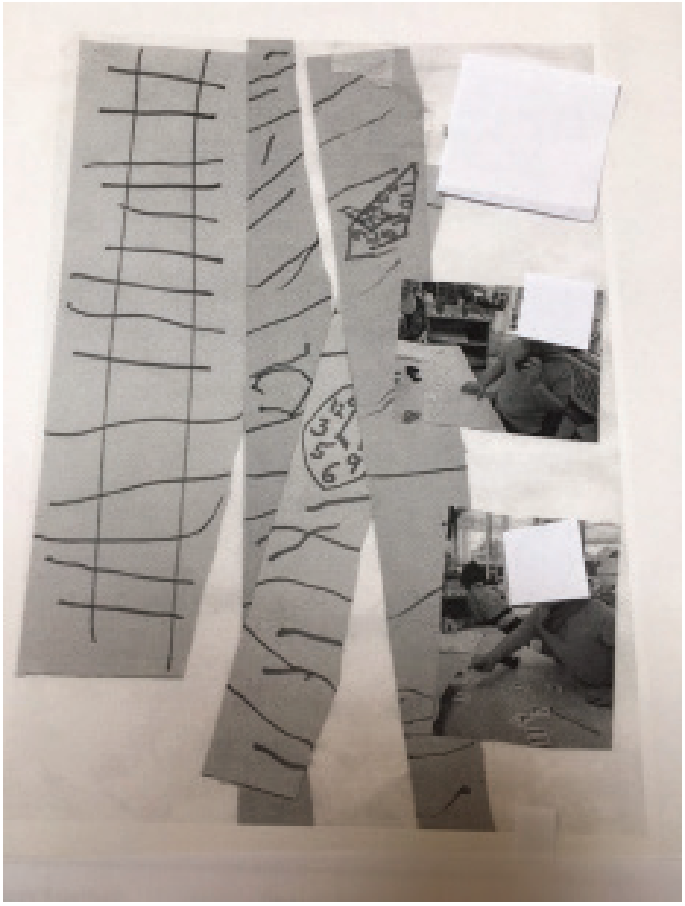
*Making a watch for superhero play*

Sue writes: "Ethan, wearing a superhero cape, is playing superheroes and stops to look for me". Ethan comes to find me to ask for help making a watch. He says, "we need paper and colours". I hold the paper while he cuts a strip off using a two-handed grip. "We need the lines on it now, blue lines and green lines. That way, the lines got to go that way." Ethan then draws a circle in the centre of the strip. "The numbers go in the circle. Sue, do the numbers". I say "You tell me what numbers to write". Ethan gets numbers from the velcro number strip and points to the ones he wants me to write and I write them in the circle as he says them. Ethan says, "five, seven, three, six, eight, that one (eleven), ten ….is that enough?" Ethan looks at his watch and it appears he is not satisfied. He says "It's not right, it's too long". Ethan starts again and calls out numbers for me to write "Ten, nine eight, seven, six, five, four, three, two, one, blast off!". As Ethan is engaged in making the watch, an argument develops between Ethan and Sammy, a nursery child and friend of Ethan. Sammy, who is working at the table beside Ethan, spreads some postcards over the table. Pointing to the writing on the postcards,

he says "this has got numbers". Ethan replies, "they are not numbers, they are letters". Sammy says, "no, they are numbers, a, b, n, s". Ethan says, these are letters. Ethan takes one of the watches and he asks me to puts it on his wrist (I do this quickly with Sellotape). He dashes off outside and says "I have super powers" as he looks down at his watch (Figure 1).

I have extended the opportunity, in this play, by introducing Isaac to the classroom clock and also a visual time line of the session. He now anticipates or estimates how much time he has to play before lunch and also at the end of the session before he goes to tea-club. "*The thing that happens before my mummy comes*". He has used his mathematical skills and learning to meet his need to know when his parents come to collect him, so regulating any anxiety he feels about waiting and therefore he is becoming more independent in his emotional well-being.

**Figure 1**



*A puppet show*

Esme writes: "I was invited to play by a small group of children who were pretending to have puppet shows, they became focused on the times their shows started and told me I was a member of the audience". Ben represented a clock on paper to use as a symbolic tool to support the evidence within the play of the time (Figure 2). This prompted Tiana to represent her very different clock (using the letters in her name to represent the numbers on the clock) which she also referenced when talking about the time of her show (Figure 3). A problem culminated when Tiana said her show did not start until four and Ben said his clock was broken and "all the numbers have gone wrong, it's gone round really fast now it's stopped, it stuck down here" he said to me. "It's stuck at half past" I replied. Ben said "yes half-past and it's run out of electricity and magic". Ben collected more 'electricity' and pretended to open his clock by turning his paper over, he pretended to insert more electricity into the 'wires' of the clock. He said he was now able to reset his clock.

In subsequent imaginary play, Tiana and Ben have revisited clocks and time, a pivotal issue in reality, which children are acutely aware of. Recent problems have led to one hand of the clock moving anti-clockwise and the other clockwise, thus enabling the possibility to go back and forward in time a response to the real problem of not being able to do this.

**Figure 2**

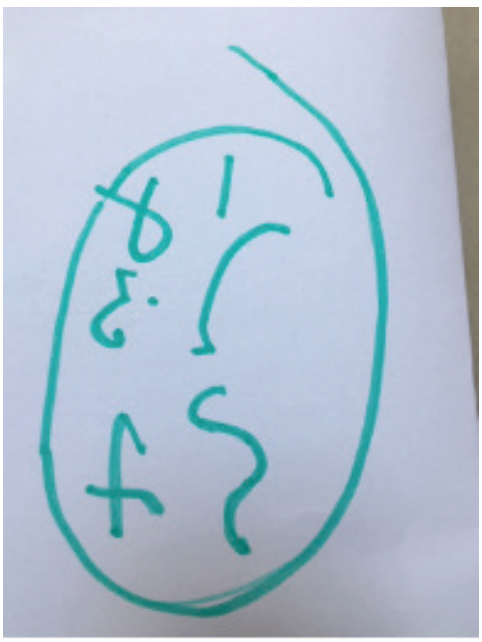
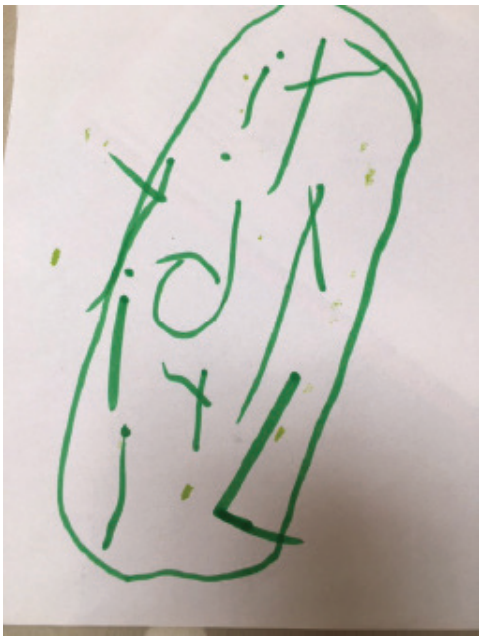


**Figure 3**



## Emerging themes and analysis

### *Children leading their play*

The initial start to the adult being with the child in these two nursery schools, in many cases, starts with the child asking the adult to be involved and this is exemplified in the two vignettes. The children are leading and are giving the teacher a role within their play. Esme is asked by the children to be involved in their play. Sue is asked to help with the resources for play and the child explicitly tells her what resources are needed. The power immediately shifts from teacher to child within the nursery play context. This is vital, as it enhances the children's meta-cognition (Whitebread, 1999), the children are active within the learning process and are not passive receivers of knowledge (Vygotsky, 1978). More importantly, the teachers except and encourage this role. For example, when Ethan asked Sue to write down the numbers on the circle he made for his watch, she asked him to tell her what numbers to write, again putting the thinking back to him. This also confirms to the child he has useful knowledge within the learning situation at nursery school. The children have also chosen the space to play, the materials and the process. In both vignettes the children are participating in 'shaping the

pedagogical' practices (Rogers, 2011, p.16). The teachers provide and environment with easily accessible resources for play and useful mathematical references, for example, number lines and clocks.

### *Noticing children's mathematical knowledge within play*

The nursery teachers highlighted the mathematics within the vignettes showing children have a connection with time and know parts of the concept and are able to use their knowledge. For example, they know watches are needed to inform you of the time. Symbols (numbers but one child used letters) are used on time equipment to represent the time of the day. During the day, times in the clock and watches are linked to certain happenings in the day e.g. home time. In Esme's vignette the importance the children placed on the time of their performance meant they understood the context of using time beyond set nursery times. Children played with the idea of time going backwards as well as forwards. They knew some of the language of time e.g. o'clock and half-past and they used these in context. Teacher's awareness of mathematics within play contexts has often been seen as lacking (Munn & Schaffer, 1993; Pound, 2006). It is vital to notice the mathematics and this needs teachers who listen generously, tuned into the possibilities of mathematics (Anantharajan, 2020; Carruthers & Worthington, 2011). However, within pretend play I argue teachers need to go further and accommodate the children's new mathematical inventions as in Ben and Tianna's two-way clock.

### *Understanding the power of the play*

In both vignettes children are trying to understand time in different ways and each with different problems. In Esme's vignette mathematical learning seems so much broader than the maths encounters expected in the standard curriculum, especially when she described that the children were thinking about going forward and backward in time. The nursery children are encouraged to use their imagination and to go as far as they want to, this could be argued to be really embracing a complex concept like time or alternatively seen as vague and without structure. The pretence affords the children the possibilities of thinking time can go back. Esme, the teacher takes this on board and appreciates the intellectual learning of the situation as she seeks to learn more about the children's mathematics and encourages the children's continual interest in broader areas of time, rather than just telling the time. Jordan (2010 p. 99) explains "Unfortunately in the absence for many teachers of sufficient knowledge, or interest in learning more, children are exposed to their teacher's reinforcement of lower level concepts such as colour, counting and shape". Teachers knowing and understanding the high-level function that is possible in pretend play (Vygotsky, 1978) is essential to embrace children's mathematical thinking. Esme was deeply in the free flow of the play whereas Sue was supporting child-initiated learning with the purpose that Isaac would

eventually use the watch he had made in his superhero play. Esme provided a running commentary as she repeated what Ben said for clarity. She was an interested player, engaged and sought to understand the meanings in the children's play as she describes in her reflections, "When observing or joining children's play, from their inventions, I notice children flexibly move from reality to fantasy, in addition to naturally drawing upon multiple mathematical positions". Esme appears secure in her support and value of play not only in a theoretical way but she observes and participates confidently with the children in their play scenario. Scrimsher & Tudge (2003, p. 298) talk about 'highly interactive relations, involving all participants in creative active growth'. Esme's descriptions of pedagogy, in particular, came into that domain of not only highly interactive relations but exceedingly intellectual connections with the children.

### *Knowledge of children's mathematical graphics*

This pretend play afforded the children opportunities to use their own mathematical inscriptions which became a vital source of their mathematical thinking. These inscriptions on paper of watches and clocks became objects for their play. The clock, in the puppet show, went beyond a drawing on paper as the children transformed (Pahl, 1999) their paper drawings to a real object within the pretend play scenario. Artefacts and signs emerge within social interaction (Kress, 1997) which children internalise because they have cognitively constructed them. Goldin and Kaput (1996, p. 415) state "internal imagistic representation is essential to virtually all mathematical insight and understanding". These graphics are also central to their mathematical problem solving and added to the meaning of their play (Carruthers & Worthington, 2011; Worthington & Van Oers, 2017). The problem for Ethan was the making of the watch and the recall of numbers and how you write them. It was important to him for his play that he had the correct written numerals and the right kind of watch. For Ben and Tiana the problem was the timing which Ben took in his own direction and drew a clock and then Tiana drew her clock, perhaps taking the idea from Ben. The teachers were listening to the children's own thinking through their graphics and accepting, but perhaps not always understanding all the idiosyncrasies that brings. They did not seem pressurised by teaching objectives although the children were involved in school curriculum areas, for example number and measurement. They were exceeding standard learning expectations for their age group; Ethan was recognising and going beyond numbers to ten (DfE, 2017) and Ben was also engaged in concepts exceeding the set curriculum by talking about time going backwards and resetting clocks.

Within Esme's reflections, it can be seen, she understood the significance of the children's graphics as symbolic tools, not just fine motor skills or marks, but meaningful inscriptions (Carruthers & Worthington, 2006; Worthington & Van Oers, 2017). Having this knowledge about children's mathematical graphics may be vital as this not only

motivates the teacher to encourage pretend play but to seek more information about children's mathematical thinking and the significance of the children's own signs and symbols.

Sue's teaching underlined the important pedagogical role of modelling mathematical symbols and signs and she did this by writing the numbers for Ethan, and by having a useful class number line reference which he used. For Esme, in her nursery, in each group area there was an analogue clock placed at the children's height and this may have provided an indirect model for Ben and Tiana's clocks.

### *Understanding children's symbolic emerging knowledge and moving beyond right and wrong answers*

In both vignettes the teachers had noted children used letters for numerals but they did not point that out to the child. Esme seems to understand children's emerging knowledge as she accepted Tiana's *different* clock, knowing children use all the knowledge they have, at the time, to represent their meaning (Carruthers & Worthington, 2006). For example, Tiana used the letters of her name for numerals because perhaps these are letters she is very familiar with and can write quickly. Children, at this age, often use letters for numerals especially in pretend play (Hall & Roberts, 2003) but it is not necessarily because they do not know the difference, instead as Tolchinsky (2003) found, children cross boundaries in informal situations. In some way, this is part of the pedagogy, not to interfere but perhaps note this for later teaching and reflection (Fisher, 2016). Going with the drift of children's play is important so as not to upset the flow (Csikszentmihalyi, 1975). Anantharajan (2020) noted the pre-school teachers in his study, in discussing and subtly noticing (Carpenter et al., 2017) aspects of children's representations developed more fine-grained understanding of children's mathematical thinking moving beyond right and wrong answers.

### *Attached teaching and knowing the child*

The nursery teachers were listening to the children at an emotional and intellectual level and this seemed crucial to enhance the mathematical development of the child. The teachers, in both vignettes, listen to the children and unlike traditional transmission teaching (Rogers, 2010) where the child tries to understand the teacher, the role is reversed and the teacher is tuning into the children's meanings. This level of knowing the child, by seriously wanting to find out, leads to attached teaching (Carruthers, 2015). The nursery teachers' knowledge of the children mostly came, not from standardised tests, but knowing the child on their terms, by listening and observing them in pretend play and knowing their contexts (Papandreou & Tsiouli, 2020). This was crucial to inform the planning of mathematics within the classroom. Sue knows Ethan well and his emotional needs and how his mathematical knowledge will make him feel more

secure, for example, in the timing of the day and knowing when his parents will pick him up. Pedagogical perspectives need to work towards a broader analysis beyond school contexts (Chieus, 2004) and this includes emotional aspects of children's mathematical lives. What is vital to the teacher child relationship is both nursery schools have embedded the Key Person approach (Elfer, Goldschmied, & Selleck, 2003). The principle of this approach is that an adult is partnered with a group of children and communicates with their families, visiting the children in their home and regularly exchanging conversations about the children with their parents or carers. The concept of the Key Person approach is usually related to the children's emotional needs (Elfer et al., 2003). However, the data has revealed it may also be relevant to mathematical development. The Key Person, as well as having an emotional connection can have an intellectual connection, with the children. This is exemplified, particularly in Esme's vignette, where she observes the children going beyond simple levels of mathematical understanding. The children generate their own problems and go beyond what they can do in reality. This helped Esme understand the children's own mathematics. The pretend mathematical play vignettes, from the nursery schools, seems to give the teachers a window into children's thinking; giving an insight into their mental life. As Day (2007, p. 1) states "teaching effectiveness is underpinned by teachers who are able to be at their best emotionally and intellectually". However, the nursery teachers' data revealed it is also vital to be *connected* emotionally *and* intellectually *with children*. I am proposing the two, intellectual and emotional attachments, could bring about a very optimum psychological sphere for mathematical learning.

## Summary

The open play, child-initiated orientation of the nursery schools was fertile ground to support children's mathematics. These two vignettes highlight equal relationships between the teacher and the child and the teachers are "conceptually and contextually connected with the children" Hedegaard and Fleer (2013, p. 56). The children are given opportunities to choose what they do, within the play environment, and they lead. Fleer (2010) writes, it is how teachers perceive children that governs how they teach them. In these vignettes, it appears, the teachers have faith in the children's ability. Children choose to use their graphics, as a tool, within this unrestricted space. Play is highly valued and this is not rhetorical but lived through everyday practice. The teachers have a significant role within children's free, mathematical, pretend play as they value and respond to the play. In this paper, some key aspects of the nursery teachers' practice are uncovered which could be the foundations of democratic pedagogies which enhance mathematical pretend play they include:

- children leading their play, which means children choose the focus of their play and organise the players, including the adults;
- time and resources being easily available for children to choose and to make artefacts or graphics for their play;
- providing useful mathematical references within the environment that children can use within their play;
- accepting and tuning into the emergent learning of children's mathematics and the mathematics of their home and community;
- being emotionally and intellectually connected to children's mathematical thinking;
- realising children have their own mathematical perspectives within the play and these might be unorthodox and very different from the standard curriculum;
- being available for the unexpected in imaginary worlds where anything can happen;
- being ready to build on children's mathematical thinking and this may be some days or weeks after the original play as they might return to similar mathematical play themes;
- understanding mathematical pretend play is complex and therefore there is a need for teachers to seek high-level professional opportunities that expand their existing knowledge of pretend play, mathematics and children's mathematical representations.

## Conclusion

The points above may be useful to teachers who want to develop a democratic play pedagogy. However, the main an overriding feature is the psychological culture which the nursery schools in these vignettes provided in making the environment respectful and truly child-led where "play is king" (Paley, 2004, p. 4) and children are confident co-creators of their own learning. Although the pretend play pedagogy reported here shows children highly engaged and leading rich mathematical learning, it is complex and is in juxtaposition to adult-led play pedagogies (Rogers, 2010) and traditional transmission teaching. In true pretend play, as exemplified in the vignettes, teachers may have to go into unchartered teaching modes and transfer into unknown spheres. This may not be an easy style of pedagogy for some early years teachers to take on, however, it can lead to young children's higher-level mathematical thinking and raises their confidence as young mathematicians.

## Limitations and further research

As stated previously, generalisations cannot be made as this is a small case study. However, the research presented here may be a useful frame for further research studies on pedagogies of mathematical child-led pretend play in environments where teachers are skilled players. There is certainly a need to enhance knowledge of mathematical pretend play pedagogy for classroom practice.

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