

‘We make stories one meter long’: children’s participation and meaningful mathematical learning in Early Childhood Education

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ABSTRACT

By taking a cultural-historical perspective, the present case study put participatory pedagogies into practice for early mathematical learning and sought to delve into children’s emerging mathematising and the teacher’s role who attentively follows children’s initiatives. Drawing on a series of selected critical incidents that evolved as the children of a kindergarten classroom engaged with the investigation of linear measuring tools, we analysed both children’s mathematising processes and teacher’s responsive and mediational acts aiming to promote young learners’ mathematical thinking. With respect to children, the analysis demonstrated how through a series of ongoing mathematising processes, they elaborated and gradually developed key measurement concepts. Regarding the teacher’s role, it was found that children’s emerging mathematising was supported by five lines of action enacted by her: (a) documenting thoroughly children’s actions, (b) posing challenging or clarifying questions, (c) introducing activities to the plenary as a response to children’s actions, (d) motivating children to ask their classmates for help, and (e) creating connections between children’s funds of knowledge and key mathematical ideas.

KEYWORDS

Participation, early mathematics, mathematising process, children’s initiatives, play pedagogies

RÉSUMÉ

En adoptant une perspective historico-culturelle, la présente étude de cas a mis en pratique des pédagogies participatives pour l'apprentissage précoce des mathématiques et a cherché à approfondir la question de l'émergence des mathématiques chez les enfants et du rôle de l'enseignant qui suit attentivement les initiatives des enfants. En s'appuyant sur une série d'incidents critiques sélectionnés qui ont évolué au fur et à mesure que les enfants d'une classe de maternelle s'engageaient dans l'étude des outils de mesure linéaire, nous avons analysé à la fois les processus de mathématisation des enfants et les actes réactifs et médiatiques de l'enseignant visant à promouvoir la pensée mathématique des jeunes apprenants. En ce qui concerne les enfants, l'analyse a démontré comment, grâce à une série de processus de mathématisation en cours, ils ont élaboré et développé progressivement des concepts de mesure clés. En ce qui concerne le rôle de l'enseignante, il a été constaté que l'émergence des mathématiques chez les enfants était soutenue par cinq lignes d'action qu'elle a édictées : (a) documenter de manière approfondie les actions des enfants, (b) poser des questions stimulantes ou éclairantes, (c) présenter des activités en plénière en réponse aux actions des enfants, (d) motiver les enfants à demander de l'aide à leurs camarades de classe, et (e) créer des liens entre les fonds de connaissances des enfants et les idées mathématiques clés.

MOTS-CLÉS

Participation, mathématiques précoces, processus mathématique, initiatives des enfants, pédagogies du jeu

INTRODUCTION

Although participation is a key factor in children's learning and is widely supported not only by researchers but also at the institutional level (e.g. curricula, documents, and reports of international organizations), recent research shows that children's involvement in meaningful learning remains a challenge for Early Childhood Education (ECE). The limited opportunities provided to children to make their own choices, pursue their interests, and take over an active role in program decisions have been identified as the most inhibiting factors for enhancing participatory learning in ECE (Kangas, 2016).

These factors constitute the major obstacles for participation during mathematical teaching in early years. Early mathematics remains trapped in two dominant trends; either structured teaching guided by the curriculum demands and the teacher's authority or occasional and fragmentary math activities, which frequently are delivered

through worksheets with vague objectives and superficial elaboration of mathematical ideas (Grossman, 2020; Ransom & Manning, 2013). As Carruthers (2015, p. 323) points out, early childhood teachers find 'mathematics a hard subject' and seem 'surprised' or 'impressed' by children's everyday mathematical knowledge when they notice it. Being confused, they are uncertain of what to do with that knowledge.

However, the direction of mathematics in ECE that considers learning through the lens of participatory and play pedagogies is recently moved forward by researchers and educators. Research evidence supports that a lot of opportunities are afforded for supporting participatory mathematical learning in ECE, provided that teachers notice children's strengths and build upon these (Dockett & Goff, 2013; Papandreou & Tsiouli, 2020; Worthington & van Oers, 2016) for inciting mathematising processes (Carruthers, 2015). In line with this pedagogical orientation, the present study sought to investigate how a kindergarten teacher supported children's deep mathematical explorations in her classroom by orchestrating various opportunities that arose during free-time activities and capitalizing on children's cultural and family mathematical knowledge.

THEORETICAL BACKGROUND

Participatory and play pedagogies in ECE

This research is based on the socio-cultural paradigm, which considers learning as joint participation in meaningful everyday activities, including play (Hill & Wood, 2019), in which the meaning is actively co-constructed by the participants through the mediation of cultural tools (Rogoff, 2008). Socio-cultural perspective of participation acknowledges children as competent and active actors (Berthelsen, 2009), while it respects their learning intentions (Hedges & Cooper, 2018). Children's participation in this study is considered as both an individual skill and a shared key competence. It is a dynamic, multidimensional and evolving process that is influenced by their lived experience in and out of school, and by the resources on which they draw upon (Berthelsen, 2009). As research has shown, children develop their co-operative and communication skills and gain more self-confidence when are considered as social actors by their teachers, while they are involved in sustained shared thinking episodes demonstrating increased autonomy, perseverance, and concentration in problem solving (Knauf, 2017; Mesquita-Pires, 2012; Papandreou & Yiallourous, 2020). As Nah and Lee (2016) point out, all these are outcomes that come from teachers' increasing sensitivity and respect for children's ideas and interests.

Research on learning and participation has recognized play as the context where children can express their knowledge, experience and interests (Hedges, Cullen, & Jordan, 2011), as well as they transform their everyday experience and achieve an elementary level of abstract thinking with the power of imagination (Vygotsky, 1978).

Seen from this perspective the dialectical relation of play and learning has been theorized through the lens of socio-cultural theory (Hedges & Cooper, 2018; Van Oers, 2010). For arguing the strong link of play and meaningful learning, Broström (2017) underlines that:

- the context of play has the power of ascribing meaning in abstract and complex concepts (e.g. when children play shopping, the concept of exchange is framed with an explicit meaning through the pretended actions of buying and selling),
- ongoing interactions and communication occur naturally when children play, which can result in high level of intersubjectivity between the teacher and young learners, and
- play sparks off children's creativity and imagination.

Contemporary play-based learning approaches are complex and demanding, but at the same time, they provide “a sophisticated blending of play, learning and teaching within participatory and relational pedagogies as a core practice of ECE” (Hedges & Cooper, 2018, p. 380). Taking into consideration the differentiated play experiences today's children have, qualified teachers observe children's play, not for recording their development, but mainly, for listening to their voices, identifying cultural and family strengths, and understanding their views (Brooker, 2011). By doing so, they are able to provide stimulating resources and make astute suggestions in accordance with children's play perspectives (Broström, 2017) and at the same time aim at their potential zone of development (Van Oers & Duijkers, 2013) “in ways that are meaningful to children” (Hedges & Cooper, 2018, p. 379).

Early mathematical learning through participatory and play pedagogies

This pedagogical orientation of ECE provides a conceptual framework for early mathematical learning that has lately gained increased acknowledgement. It exemplifies children's play or other child-initiated activities as a context where children bring and express their everyday mathematical knowledge and experience (Papandreou & Tsouli, 2020; Wothington & van Oers, 2016), while they are frequently involved in a process that van Oers (1996, p. 74) identifies as mathematising elements of play. For Jupri and Drijvers, (2016, p. 2483) mathematisation “refers to the activity of organizing and studying any kind of reality with mathematical means”, and as Kaartinen and Kumpulainen (2012, p. 265) put it, the process of mathematising “intertwines everyday knowing with mathematical tools and procedures”. This explicit link implies that early mathematization is a dialectical process and can be theorized through the Vygotskian perspective on the importance of everyday concepts for conceptual learning (Vygotsky, 1978). From this point of view, the ideas and experience that children acquire by participating in various everyday activities are considered as crucial elements for mathematising, while the concepts being developed through mathematising constitute

a strong basis for further mathematical explorations during everyday activities (Tsiouli & Papandreou, 2019).

Researchers who work on linking play pedagogies and early mathematics point out that children usually identify and use mathematical objects for accomplishing their play intentions, or play explicitly with mathematical ideas during free-time activities, adding that all these are demonstrations of emerging mathematising (Björklund, Magnusson, & Palmér, 2018; Tsiouli & Papandreou, 2019). They also argue that, being meaningful for young learners, these incidents can serve as teaching opportunities, however only if the teacher has the 'ability to seize the moment' (Björklund et al., 2018, p. 471), that is, to understand the mathematical content of children's activity and with flexible and responsive mediational acts to incite further explorations that could allow young learners to elaborate and organize their everyday mathematics. As Broström (2017, p. 10) puts it, "the idea is to establish a shared and joint interaction and to stress mutual complementarity – in short, to create an activity and relationship characterised by dialogue and intersubjectivity".

Early measurement concepts

A kind of mathematical experience that many children obtain relatively early in their lives is associated with their participation in measuring activities with rulers, measuring tapes and cups, tailor's tapes and scales, which all are artefacts that can be found in most houses (Papandreou, Sofianopoulou, Kalogiannidou, & Birbili, 2015). Being meaningful for children, these everyday activities contribute significantly in their understanding of the underling concepts (Irwin, Vistro-Yu, & Ell, 2004; MacDonald & Lowrie, 2011). But what is more important, as van den Heuvel-Panhuizen and Buys (2008, p. 10) put it, is that measurement stimulates "the development of a mathematical disposition which is characterized by an exploring attitude, a certain perseverance in solving problems, and a sensitivity to the beauty of mathematical structures and solutions".

For understanding linear measurement, children have to grasp a complex network of concepts and processes, including tiling, unit iteration and unit-measure compensation as well as the process of measuring with conventional measurement tools (Nunes & Bryant, 1996). Previous research has shown that children aged 4-6 recognize the arithmetic representation of units and the sequence of numbers on measurement tools, while some of them may have an early awareness of the similar spatial intervals depicted on tools (MacDonald & Lowrie, 2011). Besides that, children with rich measurement experience are able to describe in detail the process of measuring (Papandreou et al., 2015). However, Lowrie, Logan and Scriven (2012) underscore that although children have a considerable body of everyday measurement knowledge and experience, measuring tools have not gained yet the attention required for meaningful learning. On that basis, by providing children with various conventional tools, in this

study, we sought to investigate their spontaneous measurement endeavors along with the kinds of response to their emerging mathematizing the teacher expressed.

Purpose and research questions

Considering children's early measurement experience and knowledge as well as that mathematical learning in ECE through participatory and play pedagogies is a feasible and reasonable pedagogical direction, the present study sought to contribute to this field by exploring:

- How children's mathematizing is enacted during their spontaneous engagement with linear measurement tools?
- What are the teacher's responsive practices during key incidents of children's emerging mathematizing?

METHODOLOGY

This case study was conducted in a Kindergarten class in Thessaloniki (Greece), which welcomes children from the whole city. Altogether 15 children aged 5-6 years and their teacher participated in the project, which lasted a total of 6 months. Parents' consent along with children's agreement for video or photo recording was obtained, while pseudonyms for children's names were used during the transcription of the data.

The ECE center was selected through purposeful sampling (Stake, 2005). Our criterion was to gather rich information to obtain an in-depth understanding on the central phenomenon (Creswell, 2012). The teacher was highly qualified holding an MA degree. The pedagogical orientation of the ECE setting incorporates participatory and dialogical approaches. Free-time activities are valued and considered crucial for enabling a negotiated curriculum based on students' interests and queries. In this context, the provision of various cultural artefacts is a key factor for the emergence of children's experience, knowledge and interests (Kirova, 2010). The year that this study took place, the teacher decided to focus on children's mathematical learning by equipping the math center with extra materials, such as measurement tools from children's daily life (e.g. rulers, measuring tapes, tailor's measuring tape, folding rulers, kitchen measuring cups and weighing scales).

Research data were selected through participant observation (Creswell, 2012) employing audio and video recording, keeping field notes, drawing and taking photos. Children's contribution in this process was critical, as the teacher encouraged them to gather and post data from their explorations (i.e. drawing, photography) on a central documentation panel in order to enhance communication among them and sustain their interest on the topic of linear measurement. This process was gradually established as a common practice of the classroom and, regularly, small groups of

children commented on the documents exposed in the panel and shared their ideas about their measurement explorations.

Data was transcribed and organized according the successive phases of the project. The organized material was studied by the two authors in order to identify key incidents that included mathematising processes. The identified incidents were classified whether as children's or teacher's initiatives, while the analysis took on consideration children's emerging queries and findings, the challenges they faced to and the specific mathematical content of their enquiries, as well as the teacher's responsive and meditational acts (Table I).

RESULTS

In this section, through the presentation of the selected incidents of children's explorations (Table I), we identify the critical transitional phases between the children's and their teacher's initiatives that reflect the complex interplay between emerging and guided mathematising, which was attained through their teacher's informed decisions.

Children's initial interest in measurement tools expressed during play

After equipping the math center with various measurement tools, the teacher began to observe the children's reactions. In the first month, linear measurement tools were used informally during pretended play (e.g. the tailor's measuring tape was used as a divider for separating the wooden playhouse, the folding ruler for making unusual shapes, or for pretending measurement actions), until a child brought in the classroom a large book that unfolds lengthwise. The book impressed the children and became the critical incident that motivated them to focus on linear measurement, as some of them impulsively decided to 'make stories one meter long' as they said. Thus, the first query that arose was how to measure lengths (i.e. stories in that case) longer than the tailor's measuring tape or the folding ruler (i.e. one meter long). The emerging need for a bigger tool was addressed by making new improvised tools. The five girls, involved in this initiative, constructed three different tools, using the conventional tools of the math center as models (i.e. a ruler, the wooden folding ruler and a sewing tape).

The first one was made by Zeta and Chara from a paper strip, long enough, on which they wrote successive numbers beginning from one ('1'). On the other side, Andriana, keen to make a folding ruler, used some numbered paper strips fastened together with a piece of ribbon which passed through the holes made at the two edges of the strips. Finally, Mary made a tailor tape with a pink ribbon, in which she drew horizontal lines, while in the spaces between the lines she successively copied the numbers depicted on the formal tape. Using later her handmade folding ruler, Andriana found that "this is bigger (meaning longer) than the wooden measure, since the numbers did not fit

and the squares (i.e. the paper strips) became bigger”, concluding that it is not fair, “the measures (i.e. the measuring tools) must be the same”.

TABLE 1

The sequence of children-initiated and teacher-initiated activities

Init.	Activity context	Teacher's actions	Children's actions	Mathematising
1. C	Measurement tools are used informally during pretended play	Observes and documents children's actions	Use measurement tools not for measuring objects	
2. C	Attempts to measure lengths-objects longer than the measurement tool	Observes and documents children's actions	The 1st query emerges: How to measure lengths-objects longer than the measurement tool? Make their own tools for measurement	The tools are human constructs (Implicit)
3. T	Eliciting activity: “draw tools you can use to make measurements”	Introduces an activity, observes & orchestrates the plenary session	Express their ideas and experience with measurement tools	Acknowledging measurement in everyday activities (Explicit)
4. C	Attempts to measure distances longer than the measurement tool	Observes and documents children's actions	The 1st query is sustained A child proposes iteration of the measuring tape	The tool can be iterated (Explicit) The unit can be iterated (Implicit)
5. T	Plenary discussion and measuring various distances and objects	Introduces the sharing of children's findings to the plenary Makes connections with Emma's FoK	The 1st query is sustained The rule recurred: We have to measure with one (tool)	The need for a fixed unit of measurement emerges (Explicit)
6. C	Measuring various objects, discussing & looking for information on the internet	Observes and documents children's actions Poses challenging questions Motivates children to ask for help	The 2nd query emerged: Why the measuring tape has two different sides? Share their explanations Look for information. The 3rd query emerged	Measurements with different units result in different numerical outcomes (Implicit)

7.T	Plenary discussion and measurements with different units	Introduces the sharing of the 3rd query to the plenary and orchestrates the discussion	Discuss on the 3rd query: What are the inches and centimeters? Decide to ask for help from the technician	Measurements with different units result in different numerical outcomes (Explicit)
8.C	Measuring various objects and changing units	Observation and documentation	Transfer the new knowledge and make measurements with formal and informal units	Measurements with different units result in different numerical outcomes (Explicit)
9.C	Block play and measurements during the "Skyscrapers" investigation	Observation and documentation	Focus on measurements with different units and look for a valid measurement process	A requirement for a valid measurement is to begin from zero (Explicit)

The teacher introduces to the plenary a drawing-telling activity

Through the ongoing documentation and reflection on children's investigations after a week the teacher acknowledged that she should both sustain children's growing interest about measurement and delve deeper into their experience with measurement tools. Therefore, she initiated a drawing-telling activity (MacDonald & Lowrie, 2011; Papandreou et al., 2015), which (i.e. "draw tools that you can use to make measurements") allowed children to express the way they had conceptualized, until then, the idea of measurement.

The children drew and talked (Figure 1) about informal units (e.g. hands and feet, ribbons, iterated books in a series) and conventional artefacts (e.g. rulers and measuring tapes, a laser device used by an architect parent, sandglasses, monthly calendars, measuring cups, cooking or other scales and baby bottles), while they explained where and how we use them in everyday activities. When the teacher asked, "why are there so many different tools for measurement?", children's responses revealed their insightful and quite mature thinking on this issue: "Because each one is used for a different task", "The ruler is small and counts small things, my dad's laser can count things that are too far away and the measure doesn't reach them", "The ruler is for drawing lines, not for measuring how tall we are", "The doctor has a measure on the wall, we sit upright and we measure our height, the measuring tape does not stand on the wall, it falls".

Both children's drawings and their comments were posted on the documentation

panel next to photos taken from their previous investigations, as all of them could inspire next explorations.

FIGURE 1

	<p>Zeta: I measure with my hands and feet. This one that is like a snail is a measuring tape that you can wrap its numbers, and next to it is the glass, because we, me and my mom, measure the sugar with the glass when we make cakes. This, in the center, is a ruler and these are the glasses that we have here and measure the water (i.e. measuring cups)</p>
	<p>Emma: This is a wooden fording measure, this, my father's device. It projects a red spot on the wall, then you press the button and it shows how many meters is. Next to it is a baby bottle a ruler and the little bear with numbers on it that I have to my room, and shows how taller I became</p>
	<p>Petros: This is the diary we use to count the days at school, and this is a book. We can put a lot of books in order and see how many books measure something, 10 books, 20 books... so... This, below the others, is the measure that you pull it out and when you leave it rolls up inside and it disappears.</p>

Children's drawings and comments describing their experiences with measuring tools

Children's attempts to measure long distances

By reflecting on children's rich experience and ideas, the teacher decided to discretely follow their measurement endeavors the next days. Thus, without initiating new activities she just informed young learners that during break time they could carry the measurement box out of the classroom (i.e. a box with measuring tapes, rulers, folding rulers, and documentation tools i.e. papers, pencils, and a digital camera). Spontaneous measuring activities kept going in and out of the classroom (e.g. measuring the height of the trees, the wooden house in the schoolyard, windows etc.) with different tools, and children always noted their results on paper. However, the issue that often recurred and seemed to absorb them was "how to measure objects and distances longer than the measurement tool?"

One day, during the measurement of the outer-ramp length, the three girls involved once again pointed out that the measuring tape was not enough to cover the whole distance. Observing their attempts, the teacher seized the moment.

Teacher: Ooops! Now, how are you going to measure the ramp?

Zeta: To measure with another one.

Emma: We have no bigger measure. Let's measure with this and then we put it back where it ends.

That was the first time that the idea of iterating the measuring tool emerged. The girls iterated the measuring tape as a single unit and captured their effort with a drawing (Figure 2, left).

Zeta: it is 7, 5, 0 (she reads the number shown at the end of the measuring tape).
(Emma noted down 750).

Andrianna: Open it again to count the other half (i.e. the rest)!

Emma: How long is it;

Zeta: 2,3 and 5.

Emma noted down 235 (cm) next to her first note (Figure 2, left).

Although the girls do not proceed to add formally the two measurement results (i.e. 750 cm and 235 cm), their drawing shows their intention to join them together as a whole to represent the total length of the ramp.

FIGURE 2



Measuring the ramp by iterating a measuring tape

The teacher encourages children to share their findings to the plenary session

When the children returned to the classroom, the teacher encouraged the girls to share their finding with their classmates to the plenary session. After a short discussion, she prompted them to continue their measurements. Trying to measure a corridor from door to door, another group of children used a lot of tools to cover the distance, while the result of this measurement was the sum of the tools, that is,

“four folding rulers, the big measuring tape and a ruler”, as they said. Emma, as an expert, intervened.

Emma: It can't be so many together (*measuring tools*). We should measure with one (*tool*). My dad never does.

Teacher: What does your dad do when the distance is quite long?

Emma: My dad has a laser device! It measures the big ones ... like the previous day, he came to our classroom and measured the windows and the walls for making the map. Let's get my dad's big measuring tape... Or better the laser he has at work!

The next day Emma's father brought the laser measurement device once again. The children made a lot of measurements with both the laser device and the measuring tape, and always recorded their findings.

Children's investigation of inches and centimeters

Emma's idea about the need for a fixed measuring tool, which could be iterated preoccupied her classmates. Thus, they continued their measurements the next days without merging different tools, until they faced a new challenge, that is, the double face of the measuring tools, which include two kind of units, inches and centimeters.

Eleni: Be careful, you turned it into big numbers! (*i.e. the inches side*).

Lina: It doesn't matter.

Eleni: Yes, but it mustn't turn over. You must keep it on this side (*i.e. the cm side*).

Lina: What does it matter;

Eleni: It has other numbers on the other side.

Teacher: Why the measuring tape has different numbers on each side?

Lina: To see better! That's why they are big, you see (*Looking at the inches side*).

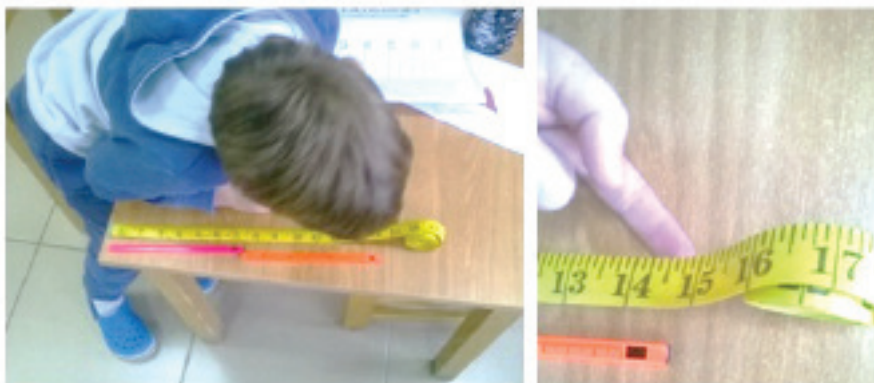
Eleni: No, it's not for seeing better!

Lina: Then why is that way?

Eleni: Because it is that way!

Without finding the wording to argue for her opinion, Eleni terminated the discussion. A day after, Kosmas aligned two pencils lengthwise and measured them, first with the folding ruler and then with a tailor tape, while he announced that “it's the same! It's 36” (*i.e., Figure 3, left*). He continued his measurement and after a while he called the teacher.

FIGURE 3



Using centimeters and inches for measuring length

Kosmas: Look at a magic trick! If I measure this way (*he unfolds the tailor tape on the centimeters side and aligns next to it the folding ruler*) they both show 36, but if I turn over this one (*he flips the tailor tape on the inches side*) it shows 15 (Figure 3, right), and now it's not the same. If I turn over both of them (*he flips the folding ruler to the inches side*) they both show 15 and it's the same again!

Teacher: Why do you think is it happening?

Kosmas was puzzled and the teacher suggested him to ask the girls who had the same query the day before. Although the day before Eleni couldn't justify her opinion, this time, referring to the marking in both sides of the tailor's tape, she constructed a complex argument that shows how much this issue had engrossed her

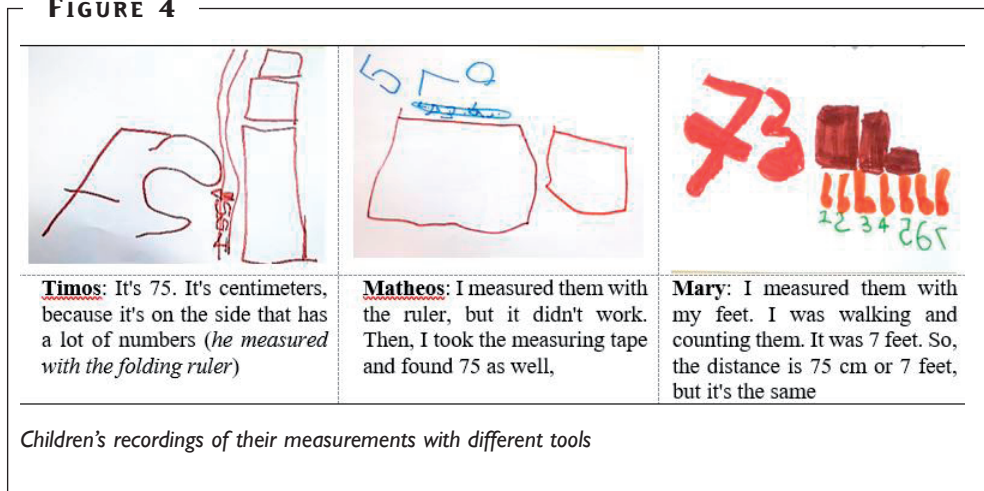
Eleni: on the one side (*she points to the inches side*), it has big numbers (*in size*), so the numbers (*the total of them on the tool*) are less, because so many numbers do not fit in; on the other side (*she shows the cm side*), where the numbers are small (*in size*), a lot of numbers fit in and so the numbers (*the total*) are more and bigger (*she means higher and shows the numbers across the tape until the number 100*).

Remaining puzzled, Kosmas suggested to look for information on the internet about inches and centimeters. However, taking the information that the measuring tapes indicate centimeters on the one side and inches on the other, a new question emerged: "what are centimeters and what are inches?"

The teacher transfers to the plenary individual challenges

The new query (3rd) was transferred to the plenary and children decided to ask the school technician for help (i.e. “He always has a measure in his bag, he will know!”). The technician compared centimeters and inches to hands and feet, explaining that, the old days, people used the later to measure. He also drew children’s attention to the reliability of centimeters and inches as fixed, standard units of measurement that everybody knows. The explanation satisfied the children, who immediately started measuring objects with different units and comparing the results.

FIGURE 4



For example, Mary, Tim and Matheos aligned three cardboard boxes and each one measured with a different tool: Mary with her feet, Tim with the folding ruler and Matheos with the ruler and the measuring tape. They spontaneously kept notes on papers to capture their observations. The teacher once again prompted the children to present their findings to the plenary (Figure 4). Mary’s comment (Figure 4, right) on their measurement results is quite interesting in that it demonstrates her understanding of how using different units ends up in different numerical outcomes but not in different measurements as the length of an object remains constant.

Children’s transfer of their new mathematical knowledge in new play activities

A few weeks later, Loukas and Andriana built two towers. Attempting to make a taller one they decided to unify the two buildings. However, moving the construction after unifying the two towers, it fell. Then they decided to rebuild it. But a new challenge arose. How would they identify which was the original tower of each one?

Loukas: Let’s rebuild them and measure your own tower and then mine. Later

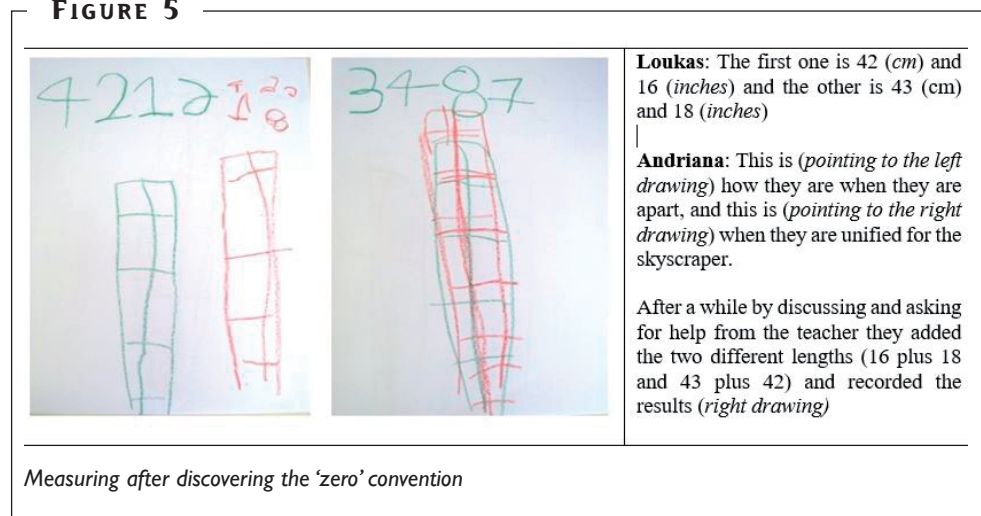
for separating them, we will know how many (*pieces*) were mine and how many (*pieces*) were yours.

In the meantime, they disagreed. Then Mary intervenes and suggests:

Mary: To be fair, we have to measure them in centimeters and inches!

They measured the two towers a lot of times, but each time the result was different. Seeking to find out what was going on, Loukas concluded: "In order to be fair, zero must be where the tower begins!". After agreeing on that *rule*, and in order to remember their measurements, they decided to record their measurements. Then they turned the paper over, redesigned their towers, the one above the other, and marked the height of the new tower.

FIGURE 5



DISCUSSION

How children's mathematising processes are enacted during their spontaneous engagement with length measuring tools?

The incidents presented in this article displayed children's successive phases of mathematising as they were unfolded during their ongoing measurement explorations mainly throughout free-time activities in their ECE classroom. As Table I demonstrates, knowing that measurement is part of various everyday activities, children realized that measuring tools are human constructs. They also became aware of the need for a fixed unit while measuring and discovered both the process of unit iteration and the compensatory principle (i.e. understand the relationship between the size of a unit and

the number of units needed to cover a distance). Considering these findings along with the process through which children's emerging mathematizing was expressed allow us to make three important remarks.

First, children do not use the mathematics they know only for serving the flow of their play scenario as previous research has shown. Besides that, they often render specific mathematical ideas and processes as 'the primary subject' of their activity (Papandreou & Tsiouli, 2020, p. 13). During the first stages of the study, the measurement instruments were used as auxiliary tools in children's play activities (Table 1, 1.C & 2.C), but later both measuring tools and measurement activities became the leading subject of children's pursuits. For example, children purposefully took the measurement box to the yard for conducting various measurements (Table 1, 4.C) or they wandered around their classroom for carrying out, recording and comparing measurements with different units (Table 1, 8.C).

Second, by interacting with each other, children created their own zones of proximal development (ZEA) (Hill & Wood, 2019). From the moment they began to use the available tools to carry out measurements, they faced a number of challenges, which represent zones for further learning of measurement concepts and processes. Co-construction of measurement meanings took place, without always having an explicit orientation. New understandings developed slowly, depending on the degree to which the children responded to the challenges they faced. Children moved between symbolic play (e.g. pretending to measure, making 'meter' stories, creating improvised measuring tools) and reality (e.g. using conventional measuring tools and performing measurements inside the classroom or to the yard). They made efforts, discussed and took decisions (e.g. "We have no bigger measure. Let's measure with this and then we put it back where it ends") or made evaluations and suggestions to their classmates [e.g. "It can't be so many together (different measuring tools). We should measure with one (tool)"]. Their reasoning and arguments evolved, while they made important discoveries [e.g. "So, the distance is 75 cm or 7 feet, but it's the same", "In order to be fair (the measurement), zero must be where the tower begins!"].

Finally, the children in this study throughout their explorations implemented a common practice of their classroom community (Wenger, 1999), that is, the recording of their measurements and using their mathematical graphics (Carruthers & Worthington, 2006) for sharing their findings with others. This practice, as the play episode in the block center demonstrated, allowed Loucas and Adrianna to deal with the arising problem and develop their mathematical reasoning. Seen from this perspective, children's mathematical graphics during play could be foregrounded as key-factor of children's mathematizing (Papandreou & Tsiouli, 2020; Worthington & van Oers, 2016).

The teacher's response to children's explorations and emerging mathematizing

Dunphy (2015, p. 297) points out that “the critical intervention of the teacher who not only recognises opportunities to encourage and support mathematisation but who proactively seeks to engage children with mathematisation processes is increasingly seen as a pedagogical imperative”.

As the present research showed, this kind of critical intervention can be offered in ECE settings in the course of free-time activities, which is in line with other researchers' suggestions (e.g. van Oers, 2010). However, our study adds a new perspective in this pedagogical orientation by revealing the strategies the teacher employed, as she sought to optimize the opportunities offered for mathematization, which we further discuss in this section.

a. Implementing long periods of documentation without intervening in children's activities

Seeking children's perspective, the teacher tried to understand the meaning they attributed to their measurement endeavors. Therefore, at the initial stages, she systematically and discreetly observed their engagement with measuring tools without intervening, since her priority was to be informed about children's measurement experience, their understanding of the measurement process and conventions. This strategy reflects a phase of preparation for considering her future potential response to children's initiatives.

b. Posing challenging or clarifying questions

Carrying on systematic observation during free-time activities and selecting critical moments the teacher used clarifying questions in order either to challenge children's thinking or to facilitate them to orally express their reasoning (e.g. “Ooops! Now, how are you going to measure the length of the ramp?”, “Why do you think is it happening?”). To our point of view, this attitude reflects what Magnusson and Pramling (2018) describe as a responsive pedagogy that respects children's play. “The key to such a pedagogy, we argue, is for the teacher to be responsive in confirming – that is, *playing along with* – the child's playful framing and within this framing pose challenging questions that ‘latch onto’ what the child enacts and says (Magnusson & Pramling, p. 38).

In the case that Eleni and Lina (Table 1, 6.C) discovered the double face of length measuring tools, the teacher posed only one question (i.e. “Why the measuring tape has different numbers on each side?”) to challenge the girls to reflect on an issue not yet explored. As we saw in this incident, the teacher did not try to sustain the discussion on that subject. As she explained in her diary, she felt that children were not ready to deal with that new challenge at that moment. Thus, she decided to give them more time. We can say that an important pedagogical strategy is to decide not only of intervening, but also of remaining

silent and doing nothing, waiting the children to revisit the same challenge or identifying another opportunity for prompting their involvement with the same issue.

c. Introducing activities to the plenary as a response to children's actions

As our results demonstrated, the teacher followed the children's initiatives and queries, without guiding their explorations. However, she introduced some activities to the plenary which were carefully planned in accordance with children's previous actions. For example, for understanding their prior experience with measurement, the teacher, except for systematic observation, introduced a drawing-telling activity, which revealed a wealth of everyday measurement experience. Besides that, after some weeks and for the rest of the project, having ensured that she had attained a good level of intersubjectivity with children, the teacher decided to bring to the plenary, occasionally, some children's measurement endeavors and discoveries being documented during free-time activities (Table 1, 5.T & 7.T). Keeping for herself the role of moderator she orchestrated those whole class discussions enabling children to share their findings, enhance their arguments, and suggest solutions. For instance, Emma argued for the process of measuring long distances [e.g. "There cannot be many together (measuring tools) my dad never does it"], while some children suggested to ask the technician for help (i.e. to explain why we use centimeters and inches).

d. Motivating children to ask their classmates for help

As the children's explorations were keeping on, mainly during free-time activities, when the teacher identified individuals or groups of children who dealt with the same query encouraged them to share their ideas and help each other. This happened, for example, when Kosmas puzzled with centimeters and inches, and after the teacher's suggestion, asked Eleni for help. In this case, the teacher's strategy gave the opportunity to Eleni not only to express her thinking, but also to extent her reasoning. As we saw, drawing implicitly on the compensatory principle, she spontaneously constructed a complex and insightful argument to explain the difference between centimeters and inches, although a day before was not able to reason on this issue.

e. Optimizing the arising opportunities to make connections with children's funds of knowledge

As Emma's experience with her father's measurement tools (i.e. the laser device and the big measuring tape) did not only emerged during drawing-telling activity but it also recurred during the whole class discussion (e.g. "My dad has a laser device! it measures the big ones", "Let's get my dad's big measuring tape... Or better the laser he has at work!"), the teacher optimized the opportunity to make connections with her funds of knowledge. She ascribed the role of expert to Emma and encouraged her to invite her father to visit the classroom.

CONCLUSIONS

Taking place into a kindergarten classroom, this case study sought to provide a better understanding of the potential pedagogies that allow everyday mathematics to flourish in ECE and enable children to guide their mathematical learning.

A part of previous research that sought to reconcile early mathematics with children's play has mainly built on teacher-initiated playful scripts in order to provide children with opportunities to develop their mathematical thinking (Ramani & Eason, 2015; Ramani, Zippert, Schweitzer, & Pan, 2014; van Oers, 1996). However, more recent research has foregrounded the meaningful mathematical play that emerges in school context and made efforts to draw teachers' attention to get involved with children's play and initiatives following children's agenda (Carruthers, 2015; Magnusson & Pramling 2018). The present study, although is in line with this pedagogical orientation, differs from other studies in that it analysed the unfolding of a series of successive children's and teacher's initiatives, seeking to identify the children's process of mathematising, and the way their teacher, following their pace, responded to them and contributed to their understanding of measurement concepts. The children in this study were clearly newcomers to formal measurement, but through their active participation in naturally occurring measurement activities were involved in successive mathematising processes. More importantly however, the ongoing mathematisation was expressed and evolved through sustained verbal and graphic interactions among peers (Papandreou & Tsiouli, 2020; Worthington & van Oers, 2016) that activated zones of proximal development (Hill & Wood, 2019) and was supported by their teacher.

Through our analysis, we have illustrated five main strategies the teacher employed for supporting children's emerging mathematising, without taking away the control of the activities from children. This orientation of the teacher's role is in line with the ongoing debate regarding teaching content knowledge in ECE through participatory and play pedagogies (e.g. Björklund et al. 2018; Vellopoulou & Papandreou, 2019). To put it differently this study, foregrounds a kind of 'child-initiated learning', which according to Carruthers (2015, p. 319) means that children choose "their own pathways to achieve their mathematical enquiries. They choose the tools and materials that will help them with their thinking and select the place where they want to carry out their enquiries".

The important point here is that children's emerging mathematising was listened, valued and seriously considered by their teacher who acted not as co-player but as stimulator, and who also enriched the classroom environment in ways that enabled children to explore and capitalize on their cultural mathematical understandings (Papandreou & Tsiouli, 2020; Worthington & Van Oers, 2016). This kind of teacher's mediation that incites and extends children's mathematising process with respect to their will and intentions constitutes what Sarama and Clements (2009) consider as 'mathematical play' and define it as a spontaneous engagement of children with math

ideas. During these encounters learning develops through the teacher's responsive mediational acts in a process where space and time were given to the child to function as an active creator of knowledge.

Although, the present study broadens our knowledge on participation in ECE, pointing to the direction of a *child-initiated mathematical learning*, it has limitations with the main one to be its narrow range as it was a case study. Future data from other classrooms and teachers would enrich the present findings and allow us to propose a framework for teachers' professional development in this field of practice.

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