Contextualization and conjecturing: elements of design of didactic sequences for the learning of Mathematics at a university level

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Abstract

In this paper we present the results of a research developed with the purpose of identifying some elements for the design of teaching sequences that promote reasoning and conjecture-based activities in undergraduate students. The methodology employed corresponds to a teaching experiment, in which the proposed activities were designed according to the theory of Mathematics in the Context of Science. The results obtained allow us to identify the relationship between the resolution process of the contextualized events and the formulation of conjectures, and to recognize the usefulness of the design of contextualized events to foster conjecturing processes for engineering students.

Key words

Conjecturing, Math modelling, Mathematics in the context of Science, differential equations, contextualized event

Résumé

Dans cet article, nous présentons les résultats d'une recherche visant à identifier *des éléments pour la conception de séquences d'enseignement favorisant le raisonnement et les activités conjecturales chez les étudiants de licence. La méthodologie employée correspond à une expérience pédagogique, dans laquelle les activités proposées ont été conçues selon la théorie des Mathématiques en Contexte Scientifique. Les résultats obtenus nous permettent d'identifier la relation entre le processus de résolution des événements contextualisés et la formulation des conjectures, et de reconnaître l'utilité de la conception d'événements contextualisés pour favoriser les processus conjecturaux chez les étudiants en ingénierie.*

Mots-clés

Conjectures, Modélisation mathématique, Mathématiques dans le contexte scientifique, Équations différentielles, Événement contextualisé

Cite this article

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Introduction

More than being a tool for solving specific problems in the engineering field, mathematics plays an important role in developing essential skills for the qualification of engineers in their professional practice (Mendoza et al., 2021; Rangel-Arzola & Ortiz-Buitrago, 2022). Within these skills, we have identified, among others, the capabilities of observation, comparison, analysis, synthesis, reflection and critical thinking (Sharhorodska et al., 2018). In this way, Cárdenas-Oliveros et al. (2022) highlight the importance that, in the mathematic courses of the undergraduate programs of engineering, these skills are promoted by means of the generation of critical thinking in students; for that matter, they could transfer mathematical knowledge to the professional development field.

According to García Barrera (2015), the development process of critical thinking in the classroom is based on the development of argumentative competences by the students. Argumentative capacity has been recognized by Guzmán-Cedillo and Flores (2020), as a fundamental skill for the working performance of future professionals. Nevertheless, it has been revealed that the argumentative capacity of undergraduate students "becomes incomplete and non-solid, because of its discursive contents, as well as the argumentative structure" (García-Barrera, 2015, p. 2). This situation occurs, among other reasons, because teachers hardly ever include activities that require the use of argumentative skills by students in their classes.

An opportunity to improve argumentative skills in undergraduate mathematics arises from activities involving mathematical modeling (Aravena et. al., 2022; Asempapa, 2018; Solar et al., 2023). Based on that, Camarena (2021) points out that, despite the fact that mathematical modeling takes part in the competences expected from a graduate student from engineering, its deployment in the mathematic courses is not completely clear, and the way in which one can develop this skill in future professionals, is not evidenced in most of the curriculums. Torres and Montiel (2020) have identified the dismantling among the mathematics courses and those of the professional circle engineering to be one of the causes of this problem.

The building process of mathematical models that represent professional duty situations of an engineer, requires the formulation of hypotheses and conjectures about the model that best adapts to the corresponding scenario (Asempapa & Sturgill, 2019; Doerr et al., 2014). In this way, different authors recognize the formulation of conjectures as an important component of the argumentative activity in the mathematics class (Komatsu & Jones, 2022; Romero & Camargo, 2022). According to Doerr et al. (2014), a learning sequence that involves the development of mathematical modeling must engage students with interpretation, conjecture, and explanation activities.

In this article, we present the results of research done with the purpose of examining the influence of the contextualized events in the development of conjecturing processes in undergraduate students from engineering programs. The investigation consisted of a learning experiment (Confrey, 2006; Molina et al., 2011), in which a didactic sequence was designed and implemented in a class of differential equations to analyze and categorize the argumentative activity of the students in the building of a mathematical model. The results of our research allow us to provide elements to structure the creation of activities for mathematics classes in undergraduate programs that promote formulation and justification processes of conjecturing in students.

Theoretical framework

This research is based on two key concepts: the conjecturing process and the contextualized events. The first one is understood as a process in which some hypotheses emerged based on diverse types of reasoning. For its part, the contextualized events serve as vehicles for transferring mathematical knowledge through authentic situations that the future engineers will encounter on their careers. Here we provide theorical description of these notions. First of all, the stages and types of reasoning that structure the conjecturing process are detailed. Then, in order to characterize the contextualized events, the principles of the theory of mathematics in context (MCS) are introduced. Finally, the importance of its authenticity as a key element for the design of didactic sequences is discussed and its purpose is to promote the formulation of conjectures in students of mathematics for the education of engineers.

Conjecturing process

Cañadas et al (2008) define a conjecture as a statement made by someone that foresees that it is veridic but that has to be submitted to a validation process. This implies that, when a conjecture is formulated, the person that does it is not totally convinced of its veracity but has some elements to state that it is correct indeed. In this way, the formulation of the conjecture is preceded by a process in which the person that nominates it is convinced of its validity, though it does not necessarily mean that the conjecture is true (Romero & Camargo, 2022). In this sense, some proposals have arisen (Cañadas et al., 2008; Astawa et al. 2018) to characterize and structure the process that leads to the formulation and validation of conjectures, which we will name here as the process of conjecture. Astawa et al. (2018) suggest five stages in order to study the cognitive process of the students in the building of mathematical conjectures. In Chart 1, we present the description of these phases and the cognitive processes that structure them.

CHART 1

Cognitive processes associated with conjecturing

Stages of the conjecturing process	**Cognitive processes developed during the corresponding stages**
Understanding the problem	To determine the information given by the problem, and what the question is. To identify the constant and variables of the problem.
Exploring the problem	To find invariant properties or patterns. To connect relevant mathematical knowledge based on the identification of properties and patterns.
Stating the conjecture	To write the conjecture generated based on the exploration process.
Justify the conjecture	To explain the reasons to formulate the conjecture based on the relation of mathematical knowledge and the related conjecture.
Prove the conjecture	Select the type of proof or demonstration that agrees to the conjecture formulated. Organize the proof.

Source: Astawa et al. (2018)

Beyond the fundamental understanding about how the conjectures are formulated and validated, Cañadas et al. (2008) have also identified five types of different conjectures that emerge during the resolution of mathematical problems, each one linked to a different reasoning way:

- *Empiric induction (discrete cases)*: A general conjecture is formulated while observing a limited number of cases and identifying a consistent pattern. This type can be typically proved through mathematical induction.
- *Empiric induction (dynamic cases)*: A general conjecture can be established through the observation of an infinite subset of continuous events by describing the nature of a dynamically related set of events.
- *Analogy*: A conjecture is established by relating a known event or a general rule to a new situation. Analogy is crucial when it comes to the professional practice of mathematics.
- *Abduction*: A conjecture emerges as a general rule in order to explain a unique event that, otherwise, would be unexplainable.
- *Perception-based*: These conjectures are formulated by a visual representation or a perceptive translation of the problem statement, in search of a new representation that clarifies the mathematical relations.

Recent investigations have been developed with the purpose of identifying principles for the design of activities that promote conjecturing processes in undergraduate students. In these investigations, the mathematical content underlined by the activities is focused on topics proper from abstract mathematics, such as geometry (Romero & Camargo, 2022), graph theory (Wardani et. al., 2019), the solution of the system of linear equations (Henriques & Martins, 2022) and the quantitative methods in the analysis of differential equations (Caicedo & Chacón, 2020). Having this in mind, we identified that the conjecturing process has not been addressed in activities that involve situations that will take part in the professional reality of undergraduate students.

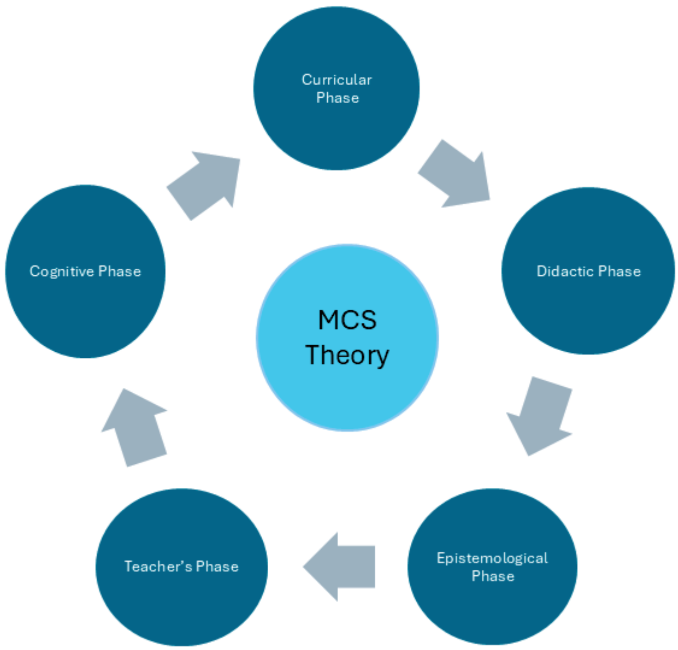
Mathematics in the context of Science

A proposal to address this problem has been developed within the theory of Mathematics in the Context of Science (MCS). This theory emerges from the research line of social mathematics, which was developed in 1982. Here, some multidisciplinary investigations have been carried out within undergraduate programs, in which mathematics is not a goal itself, but it is considered as a tool to develop mathematical competences inherent from the profession (Camarena, 2021). The MCS considers the learning-teaching process as a system in which five stages that are interrelated intervene and are presented in Figure 1:

- *Cognitive phase:* It is centered on the student's difficulties, such as their beliefs and the way in which contextualized mathematics learning is constructed, as well as identifying their prior knowledge.

- *Teacher's phase:* It addresses the teacher's professional development process in order to work with contextualized mathematics and their own beliefs.
- *Didactic phase:* It allows identifying the interaction between students and students and teachers, through the mathematics-in-context dynamic.
- *Curriculum phase:* It oversees the curriculum development of the mathematics program in accordance with the profession by ensuring coherence between the other stages and interaction among the curriculum, students, and teachers.
- *Epistemological phase*: It analyzes the knowledge that is expected from the student to build in order to offer contextualized mathematics, by establishing the relationship between mathematics and the profession.

Figure 1



Phases of Mathematics in the Context of Science. [Source: Camarena (2021)]

In our research, we focused only on the didactic phase, which serves as a reference to the design of the sequence activities. In this stage, the theory proposes the design of contextualized events, in which some situations are presented to students, whether they are from daily, professional, or working life, in which the knowledge areas of their future profession contextualize interdisciplinary mathematics. In the resolution process of these contextualized events, heuristics emerge, the thinking and metacognition skills (Camarena, 2013), which support the transferring of mathematical knowledge in the social and professional fields that require them.

The MCS theory is based on three paradigms mentioned by Camarena (2021), which correspond to:

- Mathematics is a supporting tool and a formative subject.
- Mathematics has a specific role in the superior level.
- Knowledge is born in an integrated way; that means that the student will have the capacity to make knowledge transfer from mathematics to the required areas.

According to Camarena (2021), *contextualized events* are the working tool of mathematics in context, which act as the key elements that integrate the fields. The contextualization can be supported from the other subjects in the curriculum that the student is enrolled in, as well as the knowledge areas related to future professional or working activities, or everyday life situations. These events can be of two types; on one hand, the academic events that are contextualized in the knowledge areas that the students attend during the program, and on the other hand, the real events that are contextualized in professional, working or daily life situations. These events should cause a cognitive conflict in the students as soon as they read them, but they should also engage and generate some interest, for them to continue the task to solve it.

Camarena (2021) suggests that the solving of a contextualized event is structured around eight stages, which describe the developed process by the students, from the identification of the characteristics of the event until the attainment of the solution to the questions that are posed on it. The stages proposed are:

1. To understand what is wanted from the event, that means it is necessary to identify which is the function it is destined for.
2. To identify the (dependent, independent, and controlled) variables and the constants of the event.
3. To identify the concepts and themes that are involved in the event, such as the mathematical ones, as well as those belonging to the knowledge area, field, or situation from which the contextualization is supported.
4. To determine the existing relations between the concepts mentioned in stage 3, which allows us to address the development of the mathematical model and its subsequent solution.
5. To build a mathematical model of the event.
6. To solve the mathematical model proposed.
7. To provide a solution to this event.
8. To interpret the solution of the event in terms of the fields of the context.

Contextualization and authenticity

An important space while designing the tasks that promote conjecturing processes in mathematical modeling is the contextualization of the problems exposed (Ainley et. al. 2006; Leung, 2011). In this way, Ainley et al. (2006) have identified a gap between the characteristics that present the modeling problems that are usually proposed in the classroom and the context of real-life situations that professionals face in their working life. This distance generates in students the perception that situations that are addressed in the classroom are not completely authentic to generate a genuine interest in solving them. This gap has been broadly investigated, and studies like Palm's (2008) have demonstrated that "authenticity" of problems is a critical factor that directly impacts in the student´s willingness to include considerations of the real world in their resolution process.

As for researchers like Vos (2015), authenticity in mathematical education should not be limited to the simulation or imitation of real-world activities. Vos defines "authenticity" as a social construction in which, for an aspect to be considered as authentic, it should have an origin outside of the scholar area and a certificate of originality. Based on this perspective, even if the imitations and simulations are properly designed to emulate real situations, they are still copies if they do not have a true origin or consequences in real life.

In this context, the authenticity of the addressed situations in the didactic sequences goes beyond the simulation of real problems to cover the nature of authentic practices in which mathematics is used as a dynamic tool in professional environments. As Dierdorp et al. (2011) point out, the design of educative resources inspired by these authentic practices, such as dikes monitoring or instrument calibration, has demonstrated its relevance for students, not only to understand the importance of scientific and mathematic concepts, but also to develop informal inferential reasoning that allows them to give meaning to complex data and make contextualized predictions by going beyond simplifications inherent to academic models.

However, by integrating mathematics with other disciplines such as engineering and science in working situations, as the theory of MCS proposes, it is crucial to recognize and manage the epistemological differences among the disciplines. Triantafillou et al. (2021) point out that these differences, immanent from the different ways of thinking and practicing in each field, can raise some significant challenges for the effective integration of knowledge.

In this way, it is important to consider the dynamic nature of mathematics as a tool in authentic contexts, particularly in the professional field (Wake, 2014, 2015). The authenticity of a situation, especially for future engineers, is not limited to the genuine origin or the simulation of a real problem. Besides, it implies the way in which mathematical tools are applied, adapted, and frequently redefined in the fluidity and unpredictability of real situations.

Methodology

Bearing in mind that our purpose is to analyze the knowledge of undergraduate students through the design of a learning sequence (Molina et al., 2011), our investigation settles into the research paradigm of design. According to Confrey (2006), with this type of research, it is intended to document the resources, previous knowledge, interactions, and conceptions that compromise the students in solving the activities proposed. In this way, the inquiry was developed in three stages that correspond to the design of a sequence of contextualized events, its implementation in some classes, and the retrospective analysis of the developed activities by students while working over the events.

Design of the activities

The design of the sequence of activities was based on the concept of contextualized events, which comes from the theory of Mathematics in the context of Science. A sampling of the curriculum of the different engineering undergraduate programs of the university in which the research took place was made in order to design a sequence of activities that consist of contextualized events. As a result of that research, we find out that the subject of thermodynamics is common among all the engineering programs of the institution, and it has a great number of applications that require the use of mathematics for their solving, specifically in the subject of differential equations: on the other hand, it was observed that these two subjects were settled within the same semester on the curriculum of most of the engineering programs, which allows us to identify the interdisciplinarity between both subjects.

Based on this, a sequence of activities was designed in order to develop a class of differential equations for an engineering subject. The sequence consisted of two contextualized events, which are summarized in Chart 2. These events correspond to situations in which the behavior of the temperature of certain objects is analyzed, referencing Newton's Law of Cooling. From each one of the events, two questions are posed, which aim to determine the behavior of the temperature of certain chemically interacting objects and the building of a differential equation that will represent the phenomena.

According to what is described by Camarena (2021), the events proposed can be classified by the category of the academic event in a medium cognitive level of complexity, as the contextualization source corresponds to a science that the undergraduate students attend in their undergraduate education.

CHART 2

Contextualized events proposed to the students

Event 1	Event 2
A sealed test tube that contains a chemical substance is submerged in a liquid bath, whose temperature is 20 degrees Celsius. Bearing in mind that the temperature of the chemical substance at the moment of submersion in the liquid is 40 centigrade, answer the following questions:	A sealed test tube that contains a chemical substance is submerged in a liquid bath, whose temperature is 10 degrees Celsius, in such a way that every minute, the temperature of the liquid bath is twice as much as the temperature of the prior minute. Bearing in mind that the temperature of the chemical substance is 40 degrees Celsius in the moment which it is submerged in the liquid bath, answer the following questions:
Questions: 1. How is the behavior of the chemical substance while time goes by? Does it increase, decrease or remain constant? Justify your answer. 2. Is at any moment the temperature of the chemical substance the same as the liquid bath? If the answer is affirmative, determine the exact moment this happens. 3. Pose a differential equation that represents the situation. Solve it.	

Source: Self-creation

Participants and data collection

The population in which the sequence was implemented corresponds to two groups of the subject of differential equations, belonging to the university in which the authors of this article work, which is of a private nature. The number of students in each group was around 30 people, who were enrolled mostly in the programs of chemical, mechanical, and petroleum engineering. In these programs, the subject of differential equations is settled in the fourth semester of the curriculum, simultaneously with the one of thermodynamics, in which some topics are related to the context of the problems presented in the sequence of proposed activities. The age category of the observed students is around 17 and 19 years old.

It is important to note that, until the moment of the implementation of the sequence, the students hadn't had any explanation given by the teacher of Newton's Law of Cooling in the class of differential equations. Even more, the only applications of differential equations of the first degree addressed in the course were the ones corresponding to population growth, mixes and eclectic circuits.

The sequence of designed activities was implemented in two class sessions of each group. Each one of the activities was developed in a two-hour session, in which the students solve questions included in the activity in working groups of 3 to 4 students. The sessions were oriented by the teacher of the subject, who is also one of the researchers, while another researcher was present to observe and document the interaction of the students during the development of the activity. To recollect the information, the discussion that was presented among the students during the solving of the activity was recorded. Additionally, they were asked to report the answers on a document that would be delivered to the teacher at the end of the course.

Data handling and analysis

In order to analyze the qualitative data, the audio recordings of the student's interactions were verbatim transcribed. Along with the written productions of students in the working groups, these transcriptions constituted the raw data of the research. The transcriptions were carefully prepared, and in order ensure participants anonymization, some pseudonyms were assigned and some minimum adjustments to improve readability were made, but always keeping allegiance to the original dialogue. The initial handling and organization of data were manually done in order to create "research data", which correspond to the interaction sessions presented between students, in which the way through some of the stages of the conjecturing process described by Astawa et al. (2018) was evidenced, and then were subsequently submitted to the codification process.

The codes were developed by means of a mixed framework: predefined codes (the ones based in Astawa et al. for the conjecturing stages and Cañadas et al. for the reasoning types) and emerging codes that were identified during the analysis. The codification of the data was created based on Chart 3, in which the corresponding information to the conjecturing stage that students went through in every relevant scenario was documented, as well as the characterization of the reasoning type developed by them. The initial codification of data was made by one of the researchers, who was also a teacher of the subject. The other researchers took part in the observation and complementary recollection of the student's interaction during the activity. The validation of codes and their application was carried on by discussions and agreement among researchers. The disagreements or divergent interpretations were solved by means of reflexive dialogue and collective revision of the fragmented data, ensuring the consistency and reliability of the process. The codes were iteratively refined as the analysis advanced by adjusting their definitions and application criteria in order to better capture the particularities observed.

Chart 3

Analytical tool

Identifier of the episode:	
Stage of the conjecturing process	**Reasoning used:**
To understand the problem	
To explore the problem	
To create the conjecture	
To justify the conjecture	
To prove the conjecture	

Source: Self-creation

During the analysis, we specifically focused on the identification of situations in which students were going through the initial stages of comprehension, exploration, and conjecturing formulation, due to the fact that the principal objective was to analyze the way in which the design of contextualized events could enhance the creation of mathematical hypotheses by students, considering that these two stages are essential for the development of mathematical reasoning in innovative contexts. The deepening in the justification process and formal proof of these conjectures represents a complementary research line and it is considered for further studies.

In order to illustrate the application of the codification scheme, an example of how data were analyzed and categorized by using the analytic tool is presented in Chart 4. Each episode identifies itself with an episode identifier (that refers to the contextualized event and the question that was being addressed in the episode and the working group in which the described dialogue was presented), followed by the codes of the "conjecturing trust" and "type of reasoning" assigned. The example shown corresponds to the analysis of an episode in which the students of one of the analyzed groups identify the variables od the first event in the didactic sequence. This episode is addressed in more detail in the discussion of the results of the research.

Chart 4

Example of codification process

Identifier of the episode: Event I – Question I – Group I	
Stage of the conjecturing process	**Reasoning used:** Deductive reasoning It is evidenced when students apply general theorical knowledge (Thermodynamic laws, Zero Law, Law of Heat Transfer) in order to interpret the specific situation of the problem and justify the temperature's behavior. César an Erika try using known physics laws in order to explain what is going to happen with temperature, going from a general principle to a particular situation.
<u>To understand the problem</u>: because students are actively identifying the characteristics and magnitudes of the problem, as the variability of temperature and its relation to thermic equilibrium. Their discussions revolve around how the temperature of the substance behaves in the context of a liquid bath and the application of thermodynamic principles.	
To explore the problem	
To create the conjecture	
To justify the conjecture	
To prove the conjecture	

Source: Self-creation

Trustworthiness of the research

The qualitative nature of the research implied the importance of having in mind some considerations to ensure the rigor and trustworthiness of the research. In order to do so, different strategies were adopted aligned with the criteria proposed by Lincoln et al. (1985), which address key aspects in order to ensure trustworthiness, such as credibility, transferability, dependability and confirmability.

According to Stahl and King (2020), credibility refers to the way in which the findings of qualitative research can be identified as being coherent among themselves. In other words, research is classified as trustworthy if the consistency of what is extracted from the data, allows one to recognize that the findings show the reality observed. In our research, that coherence is addressed by means of different triangulation processes. According to several researchers (Shenton, 2004; Stahl & King, 2020), the use of different information sources allows the direct strengthening of the credibility of our findings, as it provides multiple confirmation points. As mentioned in the process description for the collection of data, the transcriptions of students' interactions, as well as their written productions during the solving process of the activities, were considered. On the other hand, the fact that there were two researchers in charge of the data's analysis process, allowed them to guarantee the investigator triangulation, which enhanced the interpretative objectivity and the consistency during the recognition of patterns.

The reproducibility of the studies, particularly the ones with a qualitative approach, has been a point of contention among researchers on education (Shenton, 2004). This is mainly because of the contextual nature and the inherent interpretation that defines this type of research. Hence, instead of the literal replication, the transferability of findings is prioritized to similar contexts or, in some cases, the creation of theories or concepts that can be applicable in a wider way (theorical or analytical generalization), in contrast to the statistical generalization typical of the qualitative research. In the particular case of our study, the detailed description of the context presented in the section is crucial. It allows other investigators to identify the characteristics under within which the findings could be relevant in other environments. The transferability of our research could specifically be limited to educative contexts of engineering with curricula and similar previous levels of knowledge, in which group interaction and problem contextualization are part of the teaching methodology.

Dependability, according to Lincoln et al. (1985), is a pillar of credibility in qualitative research. Different from positivist reliability, which looks for identical results in exact repetitions, the changing nature of the qualitative phenomena makes that replication inviable. Hence, reliability is achieved by constructing trust in the trustworthiness of a rigorous process of study, demonstrating credibility in it by means of methodological consistency and the use of overlapping methods. In the present study, this is addressed by the meticulous documentation of each research stage, as it is detailed in the description of the methodology: from the design of activities and content justification to the implementation procedures in class and the detailed characterization of the population. This systematicity extended itself to the data collection, by means of audio recordings and written productions of the students, and the detailed steps of the analysis, supported by specific theoretical frameworks.

Confirmability is a crucial aspect regarding the reliability of the qualitative research, seeking to ensure that the findings reflect the expectations and ideas of participants by minimizing the investigator bias and approaching a more "objective" reality. Triangulation is also used as a strategy for credibility; here is equally fundamental to promote multiple points of view that help to discern the voices of participants of the interpretations or preferences of the researcher. A key element to achieve this purpose is transparency in the process, which implies documenting how the ideas of participants guide the results. This is achieved by a proper and detailed audit trail, already described in the discussion of the dependability of our research.

Results and discussion

The analysis of the activity developed by the students while solving the sequence evidenced their way through the stages of the process of formulation of the conjectures proposed by Astawa et. al. (2018). As mentioned before, our research focused on the first three stages of this process, in which the building of a mathematical model took place to represent the temperature behavior of the objects involved in each one of the presented events. Hereunder, we present the description of each characteristic of the reasoning used by the students of one of the working groups, for the construction of the mathematical model in each one of the stages and the way in which this reasoning was influenced by the contextualization of the corresponding event.

Understanding the problem

Astawa et al. (2018) describe this stage as one that includes the developed activity by the students while identifying the characteristics of the problem that is presented to them. In the case of the sequence of events proposed, in this stage students had to establish the behavior of the temperature of the chemical substance and the liquid bath in each one of the presented situations. In particular, the first question aims to establish the variability of the temperature and the chemical substance through time.

In this way, we observe that in this stage, the first two resolution phases of a contextualized event were developed (Camarena, 2021). Essentially, the students had to identify which one of the magnitudes involved in the event was a variable, and which one was constant within the context of each problem. To identify this, the students made a transformation of the knowledge they had about the phenomena of thermodynamics so they could apply it to the recognition of the mathematical characteristics of each event. This transformation is named by Camarena (2021) as contextualized transposition, in which a transit from the mathematical knowledge previously acquired to an applied one of the situations intended to model is made.

In the developed activity of this phase, we could identify that, in the case of the temperature of the chemical substance, the students that belonged to the analyzed groups could identify this magnitude as a variable of the problem. This is evidenced in the following scenario, in which the students of the group answer the first question of the first event of the sequence:

Ana: Well, it decreases, doesn't it? It has to be at the same temperature as the bath eventually.

César: Not exactly at the same temperature, but it should be at an equilibrium temperature. This would depend on the exchange temperature constant of the substance we will work on.

Erika: But that wouldn't have to do with the Zeroth Law of Thermodynamics, which states that higher body energy is transferred to the one with a lower energy?

César: Well, the Zeroth Law is about three bodies, in which two of them are in balance, and if a third one is in balance with any of the two mentioned, then it would also be in balance with the other. So, I think that it would be the law of heat exchange posed by thermodynamics.

In the previous scenario, we observed the way in which the integrative nature of the contextualized event favored the understanding of the characteristics of the problem. The argument to justify that the temperature of the chemical substance was variable emerged from the students' knowledge about the laws of thermodynamics, which allowed the establishment of the way in which the temperatures of both objects involved in the activity interact. In this way, according to Leung (2011), we can recognize the importance of the role of the context in the construction of the mathematical concepts that are the base of the model that students are expected to build.

On the other hand, in the collected data we could observe that the context created some conflicts with what was expected for students to identify in the problem. An example of this was observed when students in the groups argued about the second question of the first event, which discussed whether the temperatures of the liquid bath and the chemical substance would eventually be the same.

César: Let's think about the second question: would at any moment the temperature of the substance be the same as the one of the bath?

Ana: Yes, because of the law of thermal equilibrium.

César: So, we would have to specify it in the first one (relating to the question), not the second one.

Ana: No, the thing is that in the first one we have to say that one of (referring to the temperature) the chemical substances decreases because heat is transferred to the bath, so the substance will lose heat.

Erika: So, in the first one, we can say that as we have two different temperatures, the higher one will decrease to the point of being equivalent to the second one, reaching the thermal equilibrium, and in the second one, we say indeed that they will reach the same point because of the same law.

Ana: Yes, but add that the temperature of the tube decreases, because that is what they are asking us.

Erika: Alright, so, regarding the second one (referring to the question), the answer would be affirmative (both temperatures would be the same).

Ana: Since the Law of Thermal Equilibrium must be applied…

Erika: The one that states that, when two temperatures of different values reach the same point, the substance would lose heat, and the bath would gain temperature, in this case.

The discussion that is presented in the previous scenario indicates that the students consider that the temperature of the liquid bath in this event is also variable, and their argument comes once again from the thermodynamic properties of the objects that are being studied. This diverts from the learning path expected in the design of the activity, in which it was considered that the students would identify the room temperature as a constant of the model.

The dialog we described previously illustrates the situation stated by Ainley et al. (2006) about the distance between the real-life situations and the mathematical models that intend to represent them. In this way, it would be wrong to state that what was suggested by students is incorrect. In fact, in similar phenomena to the one described in the event, the objects involved behave in the way the students of this group inferred. This caused the intervention of the professor, who clarified that, in order to modulate the event, it was necessary to consider that the temperature of the liquid bath was not affected by one of the chemical substances, which is constant as a result.

In the second event, the characterization of the temperature of the involved events was made by considering the discoveries made by the students while solving the first activity. This enabled them to identify that, in this case, the temperature of the chemical substance, as well as the liquid bath, was variable. Now, the fact that in this new event the temperature of the liquid bath was variable led them to consider the effect of this fact on the variation of the temperature of the chemical substance:

Erika: Well, I don't know what you think, but looking at the first one (referring to the problem), based on what I read, I think that it will decrease (referring to the chemical substance), won't it?

César: Yes, I think it will be as such. As we can see, it is the same situation as last class, but this time the liquid bath will increase to a constant temperature.

Ana: I think that initially it decreases, but it also increases with the temperature of the bath, doesn't it? Because it has to keep being in thermal equilibrium, and the one of the bath will practically increase every minute, well, not exactly, but it is constantly increasing, isn't it?

Juan: It would be like the temperature of the substance is decreasing…

Erika: And does it go up again?

Juan: And then up, once again.

Ana: With the bath, because it states that the temperature is twice the very last one every minute, that means that it is constantly increasing.

Juan: Yes, isn't it? because it reaches a point and then keeps going, so…

Ana: Yes, but it is close, isn't it?

Juan: Yes, it had to be close, but not really the same one, because it doesn't reach a point in which it is the same, no.

Erika: Alright, but it could be said then that it doesn't have a behavior as such, but the same as the other three, I don't know if I am making myself clear, like it kind of decreases at first; that means it has different behavior by time…

The last scenario evidences the fact that the students used analogical reasoning to establish the characteristics of the event. Trench et al. (2009) characterize analogical reasoning as the process of transferring knowledge from one already known situation to a new one. In this case, we could identify that the students use the knowledge of the characteristics of the first event to comprehend what occurs in the subsequent phenomena of the second event.

An aspect to highlight in this stage is the absence of arguments of a mathematical nature for the characterization of objects involved in each one of the events. In the analyzed information, we could evidence that the answers that the students gave to the first questions emerge from their perception about the event and their knowledge of the laws of thermodynamics, with no justifying of mathematical nature. Even more, the students didn't recognize the need to build a differential equation to solve the questions proposed.

Exploring the problem

In this stage, the students identified the elements that allowed them to build the corresponding mathematical model for each one of the sequencing events. During this process, we could identify the emergence of the different types of reasoning to determine the differential equation that represented each situation.

For the first event, a significant fact of the building process of the model was related to the way in which the necessity of stating a differential equation to address the problem emerged by means of the students. As described in the design of the contextualized event, the activity included a question in which they were asked to build a differential equation to answer the previous questions. However, the students recognized the importance of building a mathematical model when they were asked to determine the moment in which the temperatures of the liquid bath and the chemical substance would be the same, assuming that it would be that way effectively. This allowed us to infer that the statement of the differential equation was recognized by the students as a need to identify the characteristics of the event and not only as a demand from the activity proposed.

Regarding the type of reasoning developed by the students for the construction of the differential equations, we observed that the students used reasoning by analogy concerning the models that the students previously knew. According to Camarena (2009), these analogies generate some bonds between the new situation that they are studying and the previous knowledge that the students have acquired. That analogy was made according to a model that they had discussed in previous classes that corresponds to the model of exponential population growth. In this sense, the initial proposal of the students regarding the mathematical model responded to the following equation where the variable T represented the temperature of the chemical substance.

$$\frac{dT}{dt} = kT$$

This allows us to evidence the transference process of the knowledge that Trench and colleagues mention. In this case, the known situation corresponds to the modeling phenomena of a population with exponential growth. Regarding the classification of the conjecturing process that Cañadas et al. (2008) describe, we could identify that in this case, a conjecture by analogy was stated.

Now, the students identified the specific characteristics of each event and required a restatement of the corresponding model. In this way, the characteristics identified in the comprehension of the problem stage took them to consider the role that the temperature of the liquid bath in the stated model should have:

Erika: Is there any way to include the room temperature? Because it affects the system that we are working on. We could include it somehow, because otherwise we wouldn't be considering the room temperature or the bath one…

César: So, we could take it as ΔT, that would be the difference of the temperatures to have in mind the room one…

Érika: So, it would be the same as the temperature (referring to the one of the chemical substances) but the one of the room temperatures, or is it the other way around?

Juan: It would be the initial temperature subtracted by the room one, wouldn't it? For it to be adjusted, because if it doesn't, it would be negative then.

Ana: Which one? The initial temperature of the bath or one of the chemical substances?

Juan: The temperature of the substance is subtracted from the one of the bath.

Regarding the second activity, the students can infer that the model should have the same structure as the one of the first activity. However, he recognizes that the temperature of the liquid bath is variable and must have an impact on the corresponding differential equation:

Erika: Okay, so, we are missing the third one which consists of stating the equation that we have already said, which turns out to be the same as the Law of Cooling, doesn't it?

Ana: Yes, if we use it as a base to state the equation.

Erika: However, we must change something or add something, because we used the room temperature as if it were a constant, that is a number, but now…

Juan: Well, it changes now; the room temperature varies regarding time.

Erika: Exactly, it would be a function that meets that purpose.

César: Yes, we will have to interpret the increasing of time as an exponential expression.

Erika: Okay, but it would be around 20°C, that is the initial temperature, multiplied by the exponential one you are referring to, that is 2, because it is considered as double, elevated to time. [The expression is written as follows $A_b=20(2^t)$]. Because I am doing the times on a paper, and it matches with $20(2^0)$, and that would be the same as 20, that is the initial temperature. In time 1 it would be 40 and it would follow the same line, so we could leave that function, I guess.

In the building of the function, we represent the liquid bath, and we identify the use of another type of reasoning described by Cañadas et. al (2008), which corresponds to the process of empiric deduction from a finite number of discrete cases. According to this, the students of the group reasoned from an inductive process, in which students identify the pattern that the temperature of the liquid bath followed through time, based on the measurement of this temperature in some specific moments.

An important component of this stage is the inclusion of the mathematical content in the reasoning processes developed by the students to generate the model. The recognition of the differential equations as the relevant object to describe the associated phenomena to the events shows the mathematization of the observations made in the comprehension stage of the problem. This allows us to identify the process in this stage described by Camarena (2021) in phase number three in the solution of the contextualized event, in which the necessary mathematical concepts are included for the mathematical model. The transit by the fourth stage of Camarena's model is also evidenced in the identification of the relations that are presented between the temperatures of the liquid bath and the chemical substance when these two objects interact.

Stating the conjecture

The exploration process developed by the students in the groups analyzed led to the stating of the differential equations to represent each one of the events. In this way, we identified that the stating of the conjecture stage agrees with the fight step in the process described by Camarena (2021). Regarding the first activity, after having identified the role of the temperature of the liquid bath in the statement of the problem, the students proceed to discuss the way in which the temperature relates to the derivative:

Erika: So, having that in mind, in the differential equation we are supposed to use, we would say that the temperature decreases proportionally to time, because for it to be a constant of proportionality, it should agree on that.

Ana: That means we would take it as it was directly proportional.

Erika: No, the variables are inversely proportional according to what we had said in the statement.

Juan: No, we would say that the temperature decrease would be proportional to time.

Erika: Oh, yes, much better…

César: So, by solving that equation, what would we have?

Erika: It would be stated that the derivative of temperature, as Ana had said, is the same as the constant k because of the temperature [of the chemical substance] subtracted by the room temperature [The following is written on the paper sheet $dT/dt=k(T-T_a)$].

In the building of the differential equation for the second activity, the students considered the obtained model for the first activity that we presented in the previous discussion. As was established in the prior subsection, in the exploring stage, the students recognized that the room temperature should be replaced by the function they got from the previous inductive process developed in this stage. This led them to modify the obtained model for the first event and state the following conjecture for the differential equation that modulated the second contextualized event:

$$\frac{dT}{dt} = k[T - 20(2^t)]$$

Conclusions

The solution process of the sequence of presented activities in this paper permits us to show the relationship between the first three stages of the conjecturing process proposed by Astawa et al. (2018) and the stages to solve the contextualized event described by Camarena (2021), which is described in Chart 5. This relationship allows us to recognize the design of contextualized events as an alternative to the generation of didactic sequences that improve the use of conjecturing activities in the mathematics classes for the education of engineers.

In the discussion presented in the previous section about the results of our research, some relevant elements were identified for the design of didactic sequences that promote conjecturing processes in undergraduate programs for engineering students. One of them is the importance of contextualization of the activities that take part in the sequence. The work developed by the students on the resolution of the events is evidence that the argumentative process was favored by the fact that the activities were settled on a context close to their professional activities. In fact, in some of the stated conjectures, the primary resource for their building came from the knowledge that they had of the study field from which the conjecture was contextualized.

Another relevant aspect that we could identify in our research had to do with the type of reasoning that was used by students to argue the conjectures that were stated.

The results presented show that an important part of the mathematical model was built from analogical reasoning, in which students related the knowledge they already had about any model or phenomenon related to the situation they were studying at the moment. This allows us to recognize the importance related to the fact that the programmed activities should include elements that allow the establishment of similarities between the characteristics of the problem settled and the situations that have been previously studied.

These last considerations revealed the potential of the contextualized events to state the conjectures in students from undergraduate programs of engineering. Naturally, the design of these events requires careful thought of the authenticity of the context in which they are immersed; in this way, we identify that the activities that were designed for this research offered an authentic enough environment for students to genuinely compromise on the solution. We consider this aspect especially relevant, because, from our perspective, which aligns with the posture of Ainley et al. (2006), a learning activity should allow the undergraduate students to recognize its utility for their professional practices. From this point of view, we can conclude that the contextualized events constitute powerful and didactic resources for the development of argumentative skills in students from engineering, in the way that they can propose hypotheses to solve problems in their professional practices in an augmented and critical way.

Chart 5

Comparative chart among the conjecturing process (Astawa et al., 2018) and the resolution of contextualized events (Camarena, 2021)

Stages of the conjecturing process	Stages of the solving of contextualized events
Understanding the problem	1. To understand what is expected from the event, that means the function it is destined for, should be identified. 2. To identify the variables (dependent, independent and controlled) and the constants of the event.
Exploring the problem	3. To identify the concepts and the topics involved in the event, from mathematical topics and concepts, as well as the ones belonging to the knowledge area, field or situation from which the contextualization is supported. 4. To determine the existing relationships between the concepts mentioned in stage 3 that allow us to address the mathematical model and the subsequent solution.
Stating the conjecture	5. To build the mathematical model of the event.

Source: Self-creation

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