Physics Didactic, Affect and Conceptualization

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ABSTRACT

This paper discusses the relation between affects and conceptualization in physics didactics. We present a theoretical framework that has been used in an investigation about basic Quantum Mechanics teaching in the high school. The Path Integrals method of Feynman has been adopted as a Reference Conceptual Structure that is an alternative to the canonical formalism. The Proposed Conceptual Structure for Teaching is described and the conceptualization and its related affective aspects are analysed.

KEYWORDS

Conceptualization, Physics Didactics, affects, quantum mechanics

RÉSUMÉ

Ce travail discute les liens entre affectivité et conceptualisation dans la didactique de la physique. Nous présentons un cadre théorique qui a été utilisé dans une recherche sur l'enseignement des notions de la mécanique quantique au lycée. On a adopté la méthode du Feynman comme structure conceptuelle de référence, en étant une alternative au formalisme canonique. La structure conceptuelle proposée pour enseigner a été détaillée et on a analysé les relations entre la conceptualisation et l'affectivité.

MOTS-CLÉS

Conceptualisation, Didactique de la Physique, affectivité, mécanique quantique

Introduction

The Physics Didactic describes the process of physical knowledge reconstruction in any institution involving one or more teachers (Otero, 2006). The specific phenomena related to Physics teaching are analysed, explained and described by the Physics Didactic. The process of knowledge construction begins outside the scientific community and is influenced by a lot of constraints, which must be analysed adopting a didactic viewpoint. In turn, these constraints greatly affect the knowledge taught.

The teacher and the students integrate the class group (CG). The CG's work is conditioned by external standards –among others, pedagogical, epistemological, institutional, and political– which establish, in fact, a set of constraints for its operation. Our didactic analysis focuses on the study of the CG members' actions related to the physical knowledge construction. The constructivist framework sets a biological continuity in the process of knowledge construction (Inhelder & Piaget, 1955; Maturana, 1995). The emotions involving coexistence and cooperation (Damasio, 1994, 2005; Maturana, 1995) are a necessary but not sufficient condition to enable the construction of knowledge in the CG. There is a deep relationship between affect and cognition. Affect is not something separated of cognition; it is part of cognition, in the same way that cognition integrates the sphere of affect (Zan, Brown, Evans & Hannula, 2006). Being a cognitive and constructivist framework with profound didactic implications, the Theory of Conceptual Fields (TCF) of Vergnaud (1990, 1994, 2000) helps describe and analyse the conceptualization in physics.

Constructivism and Knowledge

We take a position that coincides with Piaget's Genetic Epistemology and Humberto Maturana's epistemological point of view. Constructivism avoids aprioristic and empiristic conceptions and denies the existence of an absolute beginning to explain the origin and the essence of knowledge (Garcia, 2000). Considering the cognitive activity is governed by a general mechanism, the constructivist theory establishes the principle of functional continuity that begins in the purely biological processes of a living being and their organization. The constructivism does not intend to give an "intrinsic definition" of knowledge because it is a process that makes sense in a social and historical context, where "levels" or "degrees" acquire significance in that context (Garcia, 2000). The constructivism rejects the subject-object duality, the Cartesian body-mind dualism and the emotion-reason dualism.

Emotions and rationality

Emotions are kinds of relational behaviour. As such our emotions guide moment after moment our doings by specifying the relational domain in which we operate at any instant, and give to our doings their character as actions. It is the configuration of emotioning what specifies our human identity, not our rational behaviour. Rational behaviour begun as a feature of the living of our ancestors with language in the use that they made of the abstractions of the coherences of their daily living as they operated as languaging beings (Maturana, 1995). But it was then as it is now that emotions specified the domain of rational behavior in which they operated at any instant. They were not aware of this then, but now we know that every rational domain is founded on basic premises accepted a priori, that is, on emotional grounds, and that our emotions determine the rational domain in which we operate as rational beings at any instant (Maturana, 1995). Usually, the human beings are not fully aware of the emotions under which they choose their different rational arguments. They are rarely aware of the fact that what guides their living are the emotions even when they claim they are being rational.

Emotions and feelings

Damasio (2005) stresses the role of emotions in the human thought. The states of the body are modified by the emotions in a way that may or may not be evident. They are automatic, and sometimes modular. We are not always aware of their consequences once they are arranged. From a biological standpoint, emotions serve the well-being and survival of our body. Emotions precede feelings, both at the time to experience an emotion as in the historical evolution (Damasio, 2005, p. 34). Emotions are functional to a complex system of vital regulation; they are designed to avoid dangers, assist the organism to take advantage of a chance, or facilitate indirectly social relations. Emotions enable the organism to respond effectively but not creatively to favourable or threatening circumstances to survival. Feelings introduce a mental alertness and reinforce the impact of emotions affecting permanently attention and memory. Thus, together with memories, imagination and reasoning, feelings make possible the production of new, not stereotyped responses. "My hypothesis is that a feeling is the perception of a state of the body at once with the perception of a certain way of thinking and thinking with certain themes" (Damasio, 2005, p. 85-86). Feelings and emotions are basic to social relations and decision-making like reasoning required in social life. The feelings that derive from positive and negative emotions are directly involved in our social experiences. It is important to emphasize that, against certain dualistic traditions; the emotions are inherently rational because they lead to the best solution in terms of survival.

Vergnaud's Ideas about Action, Conceptualization and Activity

The TCF is based on the idea of pragmatic knowledge construction. It cannot theorise about learning neither from symbolic representations nor from situations. Hence, situations and the sense of symbols must be considered bearing in mind the students' actions in situations and their behaviour organization. Then, the concept of scheme becomes extremely relevant. Vergnaud (1990) defines "scheme" as a functional dynamic **totality,** andasthe invariant organization of activity and behaviour for a certain **class of situation.** Schemes are made up of four kinds of components: **goals**, subgoals, and expectations; **rules** of action, information search, selection and control; **operational invariants (OI)** (concepts and theorems-in-action) and **inference** possibilities.

Conceptualization is part of activity organization and the scheme associated to a kind of situation necessarily involves conceptualizations. The OI are the epistemological part of schemes whose function is to recognise the objects, their properties, relationships and transformations. As regards OI, their main functions are to take and select the relevant information and infer the useful consequences for the action, the control and the subsequent information taking. The concepts-in-action, defined as objects or predicates, are neither true nor false, they are only pertinent or not. A theorem-in-action is a proposition considered true into the activity. In sciences, since a lot of theorems-in-action could be associated to a concept, then, it makes no sense to say that some people have understood certain concept. However, it does make sense to establish which theorem-in-action people are able to use into a given situation.

Without these four scheme components the structure of the activity could not be understood. It has a double characteristic: it is systematic and contingent. First, the activity is systematic for lots of situations because it is governed by strict rules. Second, it is contingent because the rules generate different activities and behaviour taking into account the parameters of situations. In the new situations, where the student has not a scheme, this characteristic of the activity is more evident. Physics situations could be regular or aleatory, especially in Quantum Mechanics (QM), increasing the uncertainty of conceptualization and, consequently, difficulting both teaching and learning.

The concept of scheme provides a theoretic reply to the knowledge construction problem. Adapting to new situations, the OI cover an essential function, when they are available in the cognitive structure to be combined and recombined or when they appear in the situation mixing with the invariants formed before. The conceptualization function is assured by the OI; the schemes are the most important psychological tool for adapting to the new and diverse.

The students' performance in situation is based on his or her implicit or explicit knowledge. As a result, it is necessary to pay attention to the cognitive development, its continuities and breaks, its obligatory ways and the kind of problems, procedures and complex representations analysing the main mistakes and discoveries. A concept does not convey meaning in terms of only one kind of situation and a situation cannot be analysed by means of only one concept. It is necessary to research into a large set of situations and concepts, classifying the kinds of relationships, the types of problems, the scheme treatments, the linguistic and symbolic representations and the concepts organising this set. The schemesorganise the students' behaviour for a given class of situations, and the action and the symbolic representation activity, especially the linguistic activity accompanying the action. Sometimes this is an internalized activity that becomes more and more important while the situations are newer, and the problems resolution is impossible without language, in particular when new conceptualizations and concepts are required. Language has communicational and representational functions. Moreover, language contributes to thought and action organization. Language and symbolic representations play an important role in conceptualization and action.

Activity is more than behaviour: behaviour is only the visible part of activity. Therefore when analysing physical and mathematical behaviour, one must look into the representational activity underlying it. The concept of scheme is essential to cover this problem. The most important part of our knowledge consists of operating actions, and they cannot be put into words easily. This is true for every domain of knowledge, and it is even truer for any person and children, as they are unable to express the knowledge they use in action. A lot of properties in problems cannot be reduced to numerical structures, nor can they be considered as linguistic or symbolic entities only. They are concepts and theorems-in-action. The implicit character of a large part of our knowledge does not mean that explicit knowledge is not operational. But, we cannot be satisfied with a theory that would consider physics only as an explicit body of knowledge.

Even when one is interested in the function of language and symbols in the development of the mind, it is necessary to identify safely which properties of the signifier represent which properties of the signified. We are aware today that words mean different things for different individuals, especially for the teacher and each student individually. Vygotski explained 70 years ago that the "sense" given to words is different from their conventional "meaning". Therefore there is a theoretical need to analyse activity and representation as composed also of OI that may be different from the meaning of words. This problem can be solved only if we accept the idea that schemes involve concepts and theorems-in-action. It is our job to identify them, together with the other components of schemes, and representation.

Emotions, Feelings and Conceptualizations in Physics Didactic

Educational institutions as systems of vital self-regulation and a propitious environment for well-being and survival

The educational institutions are human creations that should extend, at least, as a desire or intention, the mechanisms of vital self-regulation. These non-automatic mechanisms would have to contribute to our well-being and survival. The school would offer the possibility of coexisting in mutual acceptance of others to generate common domains of knowledge. This is essential for our well-being and it is also a necessary condition to cognitive development and science learning.

The CG as space of coexistence

Students and teachers integrate groups that would have to work as a space of coexistence. Coexistence in mutual acceptance is a necessary condition to knowledge reconstruction due to the fact that it allows the students be committed to their own learning. Learning is neither obedience nor repetition. As Piaget said, it is necessary a team to think.

Coexistence requires each member of the CG accepts *legitimacy of the other one* (Maturana, 1995). Acceptance, the opposite of negation, is a basic emotion to learn in a consensual domain. Furthermore, acceptance calls for the state of being alert so as not to fall into the temptation of certitude. In the classroom, a lot of actions are usually taken from the habit of certitude. Acceptance is different from tolerance. The other one is tolerated when it is assumed that he or she is absolutely wrong while we are not. I am right, he is wrong; then I admit him to be wrong. Assuming objectivity with parentheses implies being aware that we do not have a transcendental access to the truth. The other one is as legitimate as we are and its reality is as legitimate as ours even if he or she does not like us or if we suppose he or she is dangerous. If we decide to deny the other, we shall have to assume our responsibility, but it is not because we can establish that he is wrong. Living in acceptance promotes self-awareness and self-identity.

Teachers' activity in coexistence

Even though the teacher has specific responsibilities, she or he cannot assume the student responsibility for learning. Through the proposition of teaching situations and questions, teachers invite students to join a new world of shared meanings. Situations are complex tasks, in Vergnaud's words "*all complex situations could be analysed as a combination of tasks*" (Vergnaud, 1990, p. 151). The cognitive processes are functions of the situations the students face and the answers they elaborate to respond to.

There are a great number and variety of situations and classes of situations into a conceptual field. A set of classes is generated by analysing the variables of situations. This task could be more or less difficult in certain domains. On the other hand, in QM and generally in Physics the task requires classifying the great complexity of mathematics and physics models. The students' knowledge is performed by the situations they face and manage progressively. In spite of the fact that learning is personal there are strong regularities between the student and the other one. They could be identified analysing the students'actions: How do they handle the same situation? Which previous ideas about objects, concepts and relationships among them do they have? Which are the steps of knowledge construction processes?

A very important point conceiving a teaching situation is to identify the questions and the actions necessaries to respond to it. Each situation could be thought as a combination of basic relationships between known and unknown data which drive to a set of possible questions. In Vergnaud's words (1990, p. 157) "*the didactic situations are an interesting and rich staging*" because they take nourishment from psychology, epistemology and physics. A well didactic staging is based on knowing the difficulties related to the cognitive tasks involved in the situations, the faced obstacles, the set of available knowledge and its possible representations. Cognitive Psychology is essential (Vergnaud, 1990, p. 157).

It is essential for the teacher to decide, select and design the situations that he or she will present to the students; this is his or her prime act of mediation (Vergnaud, 2000). Learning depends on the students accepting the teacher's invitation. Coexistence in acceptance entails teaching taking into account the students' knowledge before questioning it instead of imposing the teacher's knowledge. When the teacher makes room for the students' activity he is teaching in coexistence, allowing them to assume their responsibility for learning. We use Vergnaud's idea of activity as a set of actions, perceptions so that we capture relevant information, OI and control mechanisms of actions. In addition, Vergnaud's idea of activity entails sharing meanings, views and knowledge in verbal communication. How can the teacher teach something to someone without understanding the activity in the situation? How can the teacher conceive the situations to teach without understanding the specific conceptual field where he or she is and the characteristic schemes related to these situations?

Students as self-constructors of knowledge

The students have the responsibility to accept or reject the teacher's invitation to study physics. The students' learning consists in answering, evaluating and deciding the course of action with the help of the teacher and the CG. The students share with the teacher the decision as to which ways to explore and with which instruments. They, too, evaluate the quality of the answer. Learning results are uncertain from a cognitive and epistemological point of view. Learning is a complex, not a linear process.

Not only is it necessary coexistence in acceptance but also consensus to construct and validate the common knowledge in the CG. This constructed knowledge is not the same knowledge constructed in the scientific community of reference. Nevertheless, it is possible to exercise certain epistemological vigilance narrowing the gap with scientific explanations. The students' activity is a process that includes perceiving, selecting and gathering relevant information, carrying out actions, making gestures and inferences, conjecturing and answering, discussing the validity of the answers with others, voicing doubts and re-using the mistakes, sharing their knowledge and meanings with others.

Emotion as basis of reason

Rationality is based on emotion. An argument, an explanation will be considered rational as long as it satisfies the criterion of rationality of who accepts it. The emotions are the condition of possibility of reason. The scientific rationality goes further into satisfying certain logical principles which are also accepted reaching an agreement by the scientific community. Science teaching at school cannot ignore that rationality is a historical construction. If teaching assumes the emotional bases of coexistence as necessary for knowledge reconstruction, it will contribute to develop institutions more compatible with a sense of well-being and cooperation.

Mistakes being "a posteriori"

Accepting the fact that mistakes are inevitable and that they are always "a posteriori" modifies our feelings about what we call mistaking (Maturana, 1995, 2001). When do we realise we have made a mistake? We become aware of mistakes when thinking about the consequences of our actions. The traditional scholar culture does not acknowledge mistakes, understanding them in the objectivity without parentheses. This illusion of certainty causes negative feelings, devaluation and discouragement in the students. Assuming mistakes as being "a posteriori" contributes to the coexistence and mutual acceptance in the CG.

Evaluation

Evaluation is a process that allows the CG to analyse what is known and who knows it. Knowledge is reconstructed and analysed by the CG taking into account the starting point. The CG will agree that someone knows when his or her actions satisfy the criteria of validity accepted by the group. Such criteria are a consensual product, being part of the public knowledge formulated and written in documents and in the joint effort activities. In these activities, teachers and students alike analyse which questions have been answered, which have not and which might be the new goals. Evaluation must be limited neither to tests nor to a moment.

Well-being, creativity and reasoning

Well-being is functional to our survival mechanism. Social and educational institutions should be extensions of the non-automatic regulation of survival mechanisms (Damasio, 2005). The well-being of the CG is based on coexisting in acceptance and learning to avoid the emotions and actions that deny the other one. Coexistence collaborates in the process of reasoning, deciding the best way of acting in a given situation at any one time. In a state of emotional well-being the cerebral activity increases in the areas linked to reasoning and creativity while the opposite happens in a state of sadness. In other words, the students' knowledge and feelings have an impact on their emotional well-being. Nevertheless, in the traditional science classroom it is natural for the students to think that "not understanding" is normal due to the fact that they can consider this fact to be a proper disability. The consequences are repetition as learning simulation, and, sometimes, discomfort and sadness, increasing incomprehension.

An Example about QM at Secondary School

The Physics teaching researches (Fischler & Lichtfeldt, 1992; Cuppari, Rinaudo, Robutti & Violino, 1997; Pessoa, 1997; Taylor, Vokos, O' Mearac & Thornberd, 1998; Pinto & Zanetic, 1999; González, Fernández & Solbes, 2000; Moreira & Greca, 2000; Osterman & Moreira, 2000; Greca, Moreira & Herscovitz, 2001; Montenegro & Pessoa, 2002; Taylor, 2003; Paulo & Moreira, 2004; Ostermann & Ricci, 2004; Hanc & Tuleja, 2005; Osterman, Prado & Ricci, 2006) and the curriculum of many countries intend to study the basic concepts of QM at secondary school (Lobato & Greca, 2005).

In Argentina, although the Physics syllabus for secondary school covers the basic concepts of modern physics, in practice these concepts are not studied. Both secondary and university teaching have forgotten that knowledge begins by questions; as a result, only answers are taught. Therefore, it is essential to focus on teaching questions and situations as complex tasks in order to teach a science alive. To that end, some conditions must be accomplished to study meaningful questions at school. These questions should have

- **cultural and social legitimacy**: the questions must be related to issues considered relevant by society
- **physical legitimacy**: the questions must be related to basic situations in Physics
- **functional legitimacy**: the questions must be related to other issues studied at

school, in physics or in other science courses (Bosch, García, Gascón & Ruiz Higueras, 2006).

QM is transformed when it is taught at a given institution; this is the well-known phenomenon of didactic transposition (Chevallard, 1992, 1997, 1999). In Physics, there are a lot of conceptual fields (Vergnaud, 1990) in which at least one Conceptual Structure of Reference (CSR) can be distinguished and recognized (Otero, 2006). When a Physics teacher invites his students to study a specific conceptual field, he or she adopts more or less explicitly a particular CSR. A CSR is a set of concepts, the relationship between them, the principles, the affirmations of knowledge and the explanations relative to a conceptual field accepted by the scientific community of reference. Our investigation rebuilds a CSR based on Feynman's Paths Integral method (1965). A detailed analysis of this CSR can be consulted in Arlego (2008). The full proposal adapting a conceptual organization for high school students can be found in Fanaro, Otero and Arlego(2007) and Fanaro and Otero (2008). The CSR adopted will be partially or fully reconstructed by a CG or by someone who tries to study it in high school, or in basic and advanced courses at university. Moreover, the researcher in science teaching needs to establish and rebuild a CRS. On the one hand, he needs to analise the knowledge living in the scientific community and, on the other hand, the characteristics, constraints and possibilities offered by the institution where this knowledge will be reconstructed.

Any attempt to reconstruct knowledge creates a different conceptual structure for the components and the relationship between them. In a more or less explicit way, each teacher of a certain group will reconstruct or select –based on an existing structure– one conceptual structure to be taught, and, in the best of the cases, he or she will invite his or her class to study it. We coined the term Proposed Conceptual Structure for Teaching (PCST) (Otero, 2006) to describe a set of concepts, the relationship between them, the affirmations of knowledge, principles and situations related to a certain conceptual field that the teacher proposes to reconstruct based on a CSR.

There are characteristic structures related to diverse conceptual fields that are alive, adapted and accepted into certain institutions. They survive for all the time because they are viable. The design, analysis and rebuilding of a PCST related to QM, viable at high school, is a specifically didactic objective. We are also interested in replicability and adaptability in similar institutions. The structures are systems (components + organization) that include key concepts, like the relationships and fundamental principles that tie them together.

When we adopted Vergnaud's ideas about concepts and conceptualization, we included both the operating and the predicative form of conceptualization. The implicit aspects of knowledge are considered by the operating invariants involved in the conservation of the forms to organize the action. This idea of concepts related to action in all their variations makes possible to build a bridge to the underlying emotions and feelings, also included in the conceptual structures. These structures are inseparable from the set of problems and situations that give sense to them. The PCST has the following components:

- **Teaching Situations**: The situations are formulated around strong, personally, socially, scientifically, and institutionally relevant questions. The answers to give are provisional, not immediate, they require a lot of time, and, above all, they do not finish in formal schooling. The situations must be developed considering the scientific knowledge, the students' knowledge and the expected learning outcomes. The students' knowledge cannot be ignored by the designed situations. They are the result of a research activity which anticipates and controls their functioning, adaptability and viability. Teaching situations have an explicit didactic intention: they carry out activities concerning physical knowledge construction held by the students and the teacher in the class. The design, implementation and validation of teaching situations are complex processes, characteristic of the research activity in Physics didactic. In these processes students' activity and teachers' activity are analysed according to a didactic framework, a cognitive framework, or both. In spite of this, it is necessary not to confuse the two ways to evaluate the obtained results.
- **Key Concepts**: These are the main concepts that must be built. They are produced in the proposed situation and without them the posed problem cannot be resolved. We assume Vergnaud's ideas of concepts. Concepts are a short list of situations, OI and referents (symbolic representations).
- **Key Questions**: The situations proposed by the teacher are complex tasks. These situations and their derived questions will be discussed by the CG. The situations resolution calls for specific concepts that will be constructed answering the questions.
- **Emotions**: Emotions are dynamic body dispositions determining our action domain (Maturana, 1995). Our conversations affect our emotions and our emotions affect our conversations. The PCST invites the students to enter upon a knowledge domain, where the denial of the other one is avoided, and an appropriate emotional dynamics to knowledge construction is built. One of the main teacher actions comes from acceptance regarding the students' knowledge, ideas, conceptions, and room for students' learning activities.
- **Actions**: They comprise three dimensions: the biological, mental and acting dimension. In the PCST we stress the last dimension. We are interested in the teacher and students' actions related to knowledge. We need to anticipate which actions are suitable for the knowledge domain that has been built. The different

meanings of these concepts flow from the system of actions related to them in every domain and situation.

- **Symbolic representations**: They refer to the external representation, verbal and not, used in language and the systems of symbols used to talk and write about scientific concepts of every knowledge domain.

The teacher and his CG will indeed reconstruct the PCST in a certain and specific institution generating the Conceptual Structure Effectively Reconstructed (CSER). The CSER is a set of concepts, relationships between them, principles and affirmations of knowledge related to a certain conceptual field reconstructed by the CG. The teacher and the students interact in conversations characterised by an adapted emotional dynamic. Every member of the CG will relate to a personal conceptual structure and a unique network of personal and private meaning. Simultaneously, the conversation in the CG will result in the students creating a network of public and shared meaning. This consensual product is also known as "the process of meaning negotiation". This negotiation process can be more or less explicit and conscientious, depending on the professionalism of the teacher, and the distance between the CSR, the PCST and the CSER.

The Didactic Sequence

Putting into effect the previous ideas, we have designed, developed and implemented a sequence based on an alternative method for teaching the fundamentals of QM for high school students focusing on Feynman's path integrals and highlighting the emergence of quantum behavior in the double-slit experiment (DSE) (Fanaro, Otero & Arlego, 2011). First, the didactic proposal is carried out in the last year of a high school Physics course. The group has thirty (30) students aged between 17 and 18. We have analysed in depth all the protocols of the CGs synthesising activities where the teacher and the students are interacting. Apart from that, we have also analysedthe students' activity, the students' replies in a final test (Fanaro, Otero & Arlego, 2007) and the results of a test related to affective aspects after the last class (Fanaro & Otero, 2008). The sequence has been repeated three times since 2006. The steps of the sequence are synthesized as follows.

DSE with small balls and electrons

First, the students imagined and predicted the results of the DSE where small balls were used. Afterwards, this experiment was simulated using the software "Doppelspalt". This software allowed the students to appreciate the impact on the screen to generate the histogram of frequencies and visualize the theoretical curve of frequencies distribution, called $I(x)$, generated by the software. Next, the students compared their predictions about the results of the experiment with the simulation results. Then, they solved a set of tasks to measure the effect on the form of the curve when the distance between the slits and the slit widths were changed. This led the group to accept the following conclusion: *"When both slits are open, the resulting curve is the sum of the individual curves, i.e., one slit open and the other one closed and vice versa"*.

After that, the students analysed the DSE with electrons instead of small balls. The simulation allowed the students to assess the shape of *I(x),* which turned out to be very different from the curve obtained with small balls. The result was inexplicable from the classical theory and the naive idea that electrons would behave like small balls.

Even though some students were unable to identify the distribution of the interference pattern observed in experiments with mechanical waves, in general, they were disturbed by the results of the simulation. This created the need to seek an explanation of the unexpected behaviour of electrons. The group accepted another key principle in the sequence: *"Although the electrons arrive in discreet units when both slits are open, the resulting curve cannot be explained as if the electrons were small balls"*.

The distribution of electrons on the screen did not follow a pattern that could be produced by the separate contribution of particles emerging from each slit. Therefore, the students were convinced that it is inadequate to consider the electrons as particles, at least in a classical sense. This new way of considering the electrons drove us to introduce the concept of "quantum system".

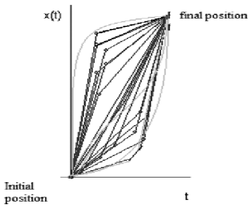
Analysis and application of SAA method for free electrons

We started by declaring that there exists a set of laws that describe objects behaviour from macroscopic to atomic scale. They are called generically *Quantum Mechanics laws.* They predict *only* the probability of an event. That is to say, given an initial state, what is the probability of arriving at a final state? In the case of the DSE the question would be: what is the probability for an electron to arrive at a given point on the screen having started from the source?

Experimentally, this probability is measured as a ratio between the number of electrons that actually reach the point and the total number of electrons emitted by the source, when the latter is very large. It is with these types of measurements that QM predictions are checked.

We have designed a sequence that emphasises the probabilistic character of the predictions as a central aspect of the quantum theory. We adopted the Feynman's method for the QM and adapted it to the students' mathematic level, calling it SAA formulation. We replaced complex numbers by two-dimensional vectors. Moreover, integrals were approximated by sums and derivatives by finite increment ratios. The method was presented as follows.

1- Suppose as initial state (I) a particle at x(t=0)=0 and as final state (F) the particle at $x(T)=x_f$. We consider here one-dimensional paths for simplicity.
Of course there are multiple forms (paths) to connect the initial state I with the final state F; some of them are shown in the following figure with straight sections (the only functions that the software used by students allows modelling).



Then, with each possible path x(t) we associate a numerical value called action, represented by "S". The action is the average difference between kinetic E_k and potential E_p energy times T.

$S= \langle E_k - E_p \rangle T,$

where<> denotes temporal average. If the particle is "free", thus it is not in the presence of forces and $E_p=0$. Then, in this case the action is simply $S = \langle E_k \rangle T$, i.e

$$S = \frac{1}{2} m \langle v^2 \rangle T$$

2- With the action S, we construct a unitary two-dimensional vector, forming an angle $S/\hbar$ withrespect to the positive x-axis. This vector is called "Probability amplitude" (A(x)) associated with the path x(t). The denominator of this quotient is $\hbar=h/2p$, where $h = 6.625 \times 10^{-34}$ Js is the so called Planck 's constant. That is to say:

Every path x(t) connecting I with F has a corresponding S, which is used to construct the Amplitude of probability vector associated to x(t), whose components are:

$$(\cos\frac{S}{\hbar}; \sin\frac{S}{\hbar})$$

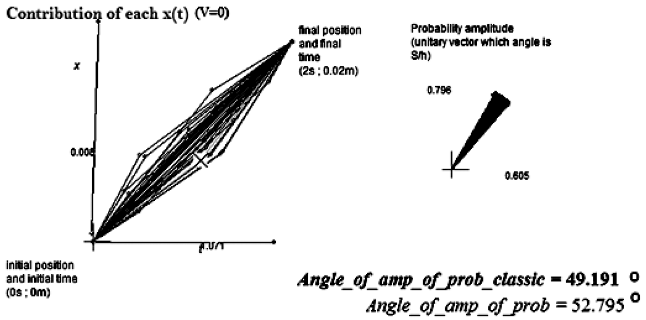
3- All amplitude of probability vectors associated to different paths connecting I with F are added. We call the resulting vector "total probability amplitude"(A_{tot} (x))$\rightarrow$

4- The square module of total probability amplitude gives the probability of arriving at final state F, having started at initial state I.

In the DSE particles can be considered as free (except on screens). We can also suppose they are sent at time intervals as long as there is no interaction with each other. The analysis of the free particle allows: a) to validate the technique, b) predict the distribution pattern on the screen, obtained in the first simulation.

To help the students apply the technique SAA to the free electron, a simulation using *Modellus* was developed. The Figure 1 shows an output screen of this simulation:

FIGURE 1

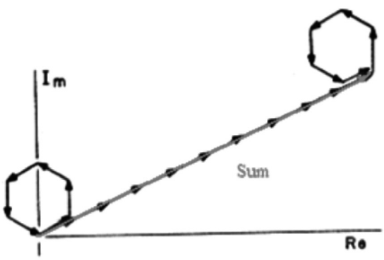


Selecting different functions x(t) that connect initial and final states, the simulation shows the angles on the Cartesian plane and the angle value of this vector in sexagesimal degrees. The probability amplitude vectors are drawn simultaneously for each function x(t) selected

The use of the simulation software *Modellus* made it possible to evaluate the contributions of different paths to the probability amplitude. It allowed students to formulate the following conclusions:

- The classical path $x_{class}(t)$ (a straight line from I to F) has the least action S.
- For atomic masses (e.g. electron mass), the angles of the amplitude vectors associated with those paths $x(t)$ near the classical path $x_{class}(t)$ are very similar. However, the angles of the vectors associated to paths x(t) which are far from the classical path are

FIGURE 2



Schematic representation of the sum of amplitude of a finite set of x (t) possible connecting the initial and final points in the case V=0. Each vector is unitary with an angle S/ℏ, and individual corresponds to the amplitude associated with one x (t) of connecting the initial and final points different from each other. This means that only a set of paths "around" the classical path contributes to the sum. The vectors associated to the paths that are far from the classical one have very different directions. They cancel each other in the sum. At this point it was emphasized that this is due to the fact that the electron is free, and that, in general, in a quantum context all paths contribute to the sum. The Figure 2 is a schematic representation of the sum for V=0.

- As particle masses increase, the contribution to the total amplitude is reduced to paths extremely close to the classical path $x_{class}(t)$. In the limiting case of a macroscopic object only the classical path contributes to the sum. In this case QM predicts the same results as classical Physics, i.e. there is only one trajectory, the one of least action.
- The transition from quantum to classical behaviour can be understood in terms of the small value of Planck's constant in a macroscopic context.

Applying the SAA method to reconstruct the interference diagram with electrons

As mentioned in the previous section, in the case of the free particle, the classical trajectory plays a central role (even at the quantum level). In fact, it can be shown that the sum over all paths can be performed analytically in this case (Shankar, 1980).

Applying these analytical results to the DSE, the probability for a particle of mass *m* to arrive at a distance *x* of second screen center, starting from the source, yields the following result (Arlego, 2008):

$$P(x) \propto 4\cos^2\left(\frac{md}{2\hbar T}x\right)$$

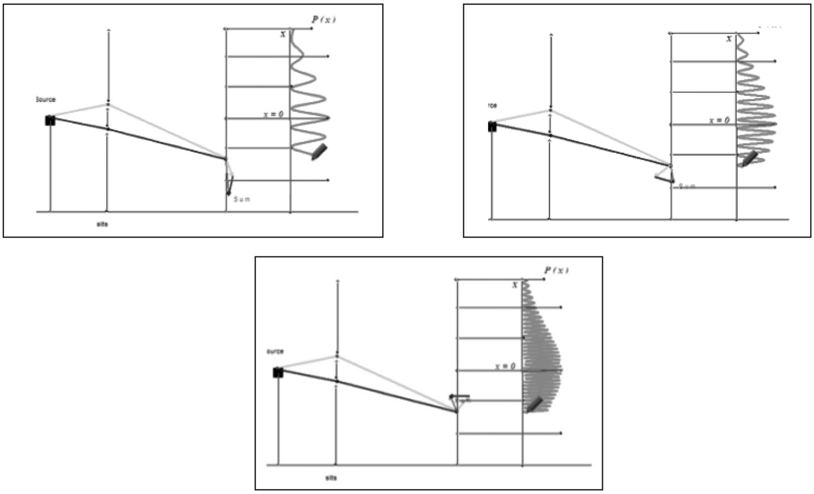
Where d is the distance between screens. Of course we immediately recognise in the previous formula the interference phenomenon. The derivation of this formula for *P(x)* was made on the basis of the mathematical level of students. However, it was emphasized that it is a direct consequence of quantum mechanics laws presented and the special role that classical paths play in the case of free particles, as themselves observed in the previous simulation.

The students discussed and analysed in groups the functional form of the expression $P(x)$ above. Using this expression with typical values of the variables provided by the teacher, they made approximate graphical representations of $P(x)$, and located maxima and minima. As a result of this analysis students recognised that these graphs adopt a similar form to graphs representing the distribution of electrons obtained in the first simulation.

Classic-quantum Transition in the DSE

A simulation with Modellus was generated to show that the ratio between the mass and Planck's constant generates, or not, the interference diagram. Fixing the rest of the parameters, it was observed how every larger value of the mass affected the P(x) curve. The software also draws the associated vector to each alternative –starting with one slit or the other– the extreme vector and the curve. The following figures show the interference diagram disappearing when the mass increases, making evident the transition between the QM and the classical mechanics.

FIGURE 3



Screens showing Double-slit Experience simulation

Coming back to the DSE phenomenon and according to the electrons arriving at the screen one at a time, the students analysed the results of the DSE obtained by Tonomura in 1974. They looked at a series of successive photographs of a collector screen.

From previous observations the students identified a wavelength (the distance between successive maxima) to be dependent on the ratio $h / (mv)$, where $v \approx d / T$ is necessary in order to give the correct units. Now, this wavelength depends only on the properties of the particle, so it makes sense to associate this wavelength to the particle itself. In this way we arrive at the concept of wavelength λ associated to the particle. It is called De Broglie wavelength, in honour to its discoverer and it is given by

$$\lambda = h / p,$$

where $p = mv$ is the momentum of the particle. From this formula it is clear that it is the ratio between m (more precisely p) and h what determines the quantum or classical behaviour of the system. For macroscopic objects λ is so small that ondulatory characteristics are imperceptible. On the other hand, for electrons, for instance, where p is of the order of h, λ is large enough to exhibit interference effects. Matter behaviour is actually well described by the Quantum Theory.

Immediately the question was put to the students why quantum interference is not detected if the experiment is realised with small balls. The students were invited to analyse the relationship between the associated wavelength and the interference diagram. Why it does not happen with the small balls while it is possible to detect it with electrons. In this last case, the quotient between Planck's constant and the mass is extremely small, due to the value of h; therefore, the associated wavelength is too small, and the maximums and minimums on the curve P(x) are indistinguishable, obtaining an average curve similar to the classical curve. The sequence finished analysing the role of Planck's constant as a fundamental constant in nature to establish if the quantum behaviour is evident or not.

The test about affective aspects

The students individually responded a test of 30 closed questions and one open questionafter the end of sequence. Each closed question offered a scale of five options between: nothing in agreement and total agreement. The coefficient alpha of Cronbach is a=0.7. The questions were related to the following aspects:

<u>a) Feelings and perceptions about the quantum concepts:</u>

In previous exploratory studies, we founded that the uncertainty of the quantum world usually produces misfortune feelings, until certain malaise; perhaps because traditionally the scholastic physics favors ingenuous deterministic realistic positions. Also we supposed that the abstraction of the quantum concepts, the impossibility to imagine them and their epistemological implications, would affect the students. We asked about:

- **Mathematical difficulty**: The majority of the students considered that mathematical aspects were accessible to its previous knowledge. Nevertheless, a little more half said that they not remembered some mathematical aspects and this was problematic understanding the quantum concepts.
- **Difficulty and disagreement:** Lots of students were surprised by the peculiar behavior of electrons; while they recognized that the probabilistic character is not comfortable for them. The quantum concepts were strange, difficult to imagine for a great majority.
- **Interest and motivation**: Half of the students say it felt interest to understand ideas of present physics. Although then the other half would feel no interest, many students felt satisfaction studying the present knowledge of physics. In addition almost everybody has valued positively the possibility of learning physics in this way.

- **Relation with its previous knowledge**: Almost all the students consider that these new concepts have relation to their previous physical knowledge. A great part of them says to feel calm because although the quantum principles are novel and surprise, also explain the classic results, that they already knew before.

b) Feelings and perceptions about the didactic sequence

- **Effort / obtained results**: Many students recognize that they had to realize a big effort to understand. Also all feel the situations were accessible and they could solve them.
- **Challenges / explanations of the professor**: Two thirds of students felt like to taste the challenges and raised questions. Almost all feel that "they would have understood more" if the teacher would have teach them, instead to face them situations and questions.
- **Individual work/ group work**: Many students recognize the relevance of group work. Also almost all felt well and necessary the individual written synthesis that they must realize at the end of the sequence.
- **Confidence in software**: students say that they trust software more than in his ideas. Like it happens with a book, it does not think that could be bad. The scholar culture is based on the request of obedience more than in the questionings.
- **Utility of the simulations**: almost all the students consider that the simulations are useful, pleasant; although they are not visually attractive they collaborate in the understanding and reduce the abstraction.
- **Effort using simulations**: the students say that working with simulations was not easy. This is related to the fact that the simulations were functional to the situations raised by the sequence, because they presented a problematic character. Besides to understand what it showed, they involved questions whose answer allows new aspects and concepts understanding.

Final Comments

The main tasks entailed in the PCST design were designing teaching situations, anticipating possible questions and answers, selecting the available software, creating simulations to visualise the SAA technique and the effects of mass increasing that were simulated with Modellus. We have described and analysed the conceptualization and its related affective aspects. It has been a very complex process to reduce and manage the knowledge of Physics in this conceptual field to make it teachable at school. It was complicated to decide which concepts and principles should be studied and how a PCST should be designed, carried out and adjusted. We consider the PCST outlined as just the beginning to discuss, modify and talk to physicists, researchers in Physics teaching and teachers. Without consultation with these three groups of actors, it would be impossible to bring knowledge alive and bridge the gap between the school and the scientific community.

The OI like theorems-in-action, the inferences and the whole activity that were identified are described in other papers (Fanaro & Otero, 2009; Fanaro, Otero & Arlego, 2009, 2011). "Conceptualization" is a long term process for it does not finish in the years of schooling. The students found out that the electrons had a special and characteristic behaviour that allowed us to think about them as quantum systems. Most of the students were unable to accept the impossibility of knowing which function would describe the electron movement. After the sequence, the students still thought: *"Finally, the electron must take some path or other"*. The students agreed that the SAA technique was a suitable mechanism to explain the interference pattern in the DSE in other ways inexplicable. Furthermore, they understood that the wave behaviour allowed to associate a wavelength not only to the macroscopic particles but also to the microscopic ones. The students related the shape and detection of the interference pattern in the macroscopic and microscopic particle cases giving a new meaning to Planck's constant. They understood its role in the quantum-classic limit.

The sequence implementation demanded a great effort on the teacher and students alike. We analysed the affective aspects in two moments: during the classes and at the end of the sequence. We used as data source the protocols of the students situation by situation and the replies in a test, respectively. The sequence has been carried out according to the predicted steps in the estimated time. The emotional dynamic of coexistence in the CG is a necessary condition for quantum concepts construction. It requires the students to make the necessary cognitive effort. They expressed on many occasions they had made an intense but possible effort. The students were not surpassed by the proposed situations and they accepted the challenges.

On the one hand, a cognitive effort was required so the students had to relate the new concepts with the previous ones, and if it was necessary, to change their usual ideas. On the other hand, an affective effort had to be carried out, feeling well facing challenges and questions and accepting that the usual ideas are wrong. Moreover, the teacher had to do a big effort, making room to students, without taking their responsibility in learning, allowing them to make mistakes and reconsider their ideas, accepting students' ideas, and waiting for students to solve problems.

Finally, we stressed the relationship between affects and conceptualization. This is a complex relation and we only have done an exploratory study. Our results are in agreement with other studies integrating cognitive and affective aspects in science teaching research (Zan et al., 2006; Machado, Frade & Da Rocha Falcão, 2010).

References

Arlego, M. (2008). Los fundamentos de la mecánica cuántica en la escuela secundaria utilizando el concepto de integral de camino. *Revista Electrónica de Investigación en Educación en Ciencias*, 3(1), 59-66.

Bosch, M., García, F., Gascón, J. & Ruiz Higueras, L. (2006). La modelización matemática y el problema de la articulación de la matemática escolar. Una propuesta desde la teoría antropológica de lo didáctico. *Educación Matemática*, 18(2), 37-74.

Chevallard, Y. (1992). Concepts fondamentaux de la didactique: Perspectives apportées par une approche anthropologique. *Recherches en Didactique des Mathématiques*, 12(1), 73-112.

Chevallard, Y. (1997). Familière et problématique, la figure du professeur. *Recherches en Didactique des Mathématiques*, 17(3), 17-54.

Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en Didactique des Mathématiques*, 19(2), 221-265.

Cuppari, A., Rinaudo, G., Robutti, O. & Violino, P. (1997). Gradual introduction of some aspects of quantum mechanics in a high school curriculum. *Physics Education*, 32, 302-308.

Damasio, A.(1994). *Descartes' error: Emotion, reason, and the human brain* (New York: Grosset/Putnam).

Damasio, A. (2005). *En busca de Spinoza. Neurobiología de la emoción y los sentimientos* (Barcelona: Editorial Crítica).

Fanaro, M., Otero, M. R. & Arlego, M. (2007). El método de caminos múltiples de Feynman para enseñar los conceptos fundamentales de la Mecánica Quántica en la escuela secundaria. *Caderno Catarinense de Ensino de Física*, 22, 233-260.

Fanaro, M., Otero, M. R & Arlego, M. A. (2009). Teaching the foundations of quantum mechanics in secondary school: a proposed conceptual structure. *Investigações em Ensino de Ciências*, 14(1), 37-64.

Fanaro, M., Otero, M. R.& Arlego, M. A. (2011). *Análisis de la conceptualización de un grupo de estudiantes de escuela secundaria a la bordar situaciones de mecánica cuántica*. Paper presented at the I Congreso Internacional de Enseñanza de las Ciencias y la Matemática, Tandil, Argentina, 8-11 de Noviembre.

Fanaro, M. & Otero, M. R. (2008). Basics Quantum Mechanics teaching in secondary school: One conceptual structure based on Paths Integrals Method. *Latin-American Journal of Physics Education*, 2(2), 103-112.

Fanaro, M. & Otero, M. R. (2009). Teoremas en acto y situaciones de Mecánica Cuántica en la escuela media. *Latin-American Journal of Physics Education*, 2(2), 24-33.

Fischler, H. & Lichtfeldt, M. (1992). Modern Physics and students' conceptions. *International Journal of Science Education*, 14(2),181-190.

García, R. (2000). *El conocimiento en construcción. De las formulaciones de Jean Piaget a la teoría de sistemas complejos* (España: Gedisa).

González, E., Fernández, P. & Solbes, J. (2000). Dificultades de docentes de ciencia en la conceptualización de temas de Física actual. En *Actas del V Simposio de Investigación en Educación en Física* (Argentina: IEF), 138-147.

Greca, I., Moreira, M. A. & Herscovitz, V. (2001). Uma proposta para o ensino de Mecânica Quântica. *Revista Brasileira de Ensino de Física*, 23(4), 444-457.

Hanc, J. & Tuleja, S. (2005). The Feynman Quantum Mechanics with the help of Java applets and physlets in Slovakia. In *Proceedings of 10th Workshop on Multimedia in Physics Teaching and*

Learning, Berlin, 5-7 October. [http://pen.physik.uni-kl.de/w_jodl/MPTL/MPTL10/contributions/hanc/Hanc-Tuleja.pdf].

Inhelder, B. & Piaget, J. (1955). *De la logique de l'enfant à la logique de l'adolescent* (Paris: PUF).

Lobato, T. & Greca, I. (2005). Analise da inserção de conteúdos de teoria quântica nos currículos de física do ensino médio. *Ciencia & Educación*, 11(1), 119-132.

Machado, M. C., Frade, C. & Da Rocha Falcão, J. (2010). Influência de aspectos afetivos na relação entre professor e alunos em sala de aula de Matemática. *Boletim de Educação Matemática*, 23(36), 683-713.

Maturana, H. R. (1995). *La realidad ¿objetiva o construida? Fundamentos biológicos de la realidad* (Barcelona: Anthoropos).

Maturana, H. R. (2001). *Cognição, ciência e vida cotidiana* (Brazil: Editorial UFMG).

Montenegro, R. L. & Pessoa, J. O. (2002). Interpretações da teoria quântica e as concepções dos alunos do curso de física. *Investigações em Ensino de Ciências*, 7(2), 107-126.

Moreira, M. A. & Greca, I. M. (2000). *Introdução a Mecânica Quântica: seria o caso de evitar a aprendizagem significativa (subordinada)?* Paper presented at the III Encontro Internacional sobre Aprendizagem Significatica. Peniche, Portugal, 11-15 Setembro.

Osterman, F. & Moreira, M. A. (2000). Uma revisão bibliográfica sobre a área de pesquisa física moderna e contemporânea no ensino médio. *Investigações em ensino de ciências*, 5(1), 23-48.

Osterman, F. & Ricci, T. (2004). Construindo uma unidade didática conceitual sobre mecânica quântica: um estudo na formação de professores de fisica. *Ciencia & Educação*, 10(2) 235-257.

Osterman, F., Prado, S. & Ricci, T. (2006). Desenvolvimento de um software para o ensino de fundamentos de Física Quântica. *Física na Escola*, 7(1), 22-25.

Otero, M. R. (2006). Emociones, sentimientos y razonamientos en Didáctica de las Ciencias. *Revista Electrónica de Investigación en Educación en Ciencias*, 1(1) 24-53.

Paulo, I. J. C. & Moreira, M. A. (2004). Abordando conceitos fundamentais da mecânica quântica no nivel médio. *Revista da Associação Brasileira de Pesquisa em Educação em Ciência*, 4, 63-73.

Pessoa, J. O. (1997). Interferometria, interpretação e intuição: uma introdução conceitual à Física Quântica. *Revista Brasileira de Ensino de Física*, 19(1), 27-47.

Pinto, A. C. & Zanetic, J. (1999). É possível levar a Física Quântica para o ensino médio? *CadernoCatarinense de Ensino de Física*, 16(1), 7-34.

Shankar, R. (1980). *Quantum Mechanics* (New York: Plenum Press).

Taylor, E. (2003). A call to action. *American Journal of Physics*, 71(5), 423-425.

Taylor, F., Vokos, S., O'Mearac, J. & Hornberd, N. (1998). Teaching Feynman's sum-over-paths quantum theory. *Computers in Physics*, 12(2), 190-199.

Vergnaud, G. (1990). La théorie des champs conceptuels. *Recherches en Didactique des Mathématiques*, 10 (2/3), 133-170.

Vergnaud, G. (1994). Le rôle de l'enseignant à la lumière des concepts de schème et de champ conceptuel. In M. Artigue, R. Gras, C. Laborde & P. Tavignot (eds) *Vingt ans de Didactique des Mathématiques en France* (Grenoble: La Pensée Sauvage), 177-191.

Vergnaud, G. (2000). *Lev Vygotsky Pédagogue et penseur de notre temps* (Paris: Hachette).

Zan, R., Brown, L., Evans, J. & Hannula, M. (2006). Affect in mathematics education: an introduction. *Educational Studies in Mathematics*, 63(2), 113-121.